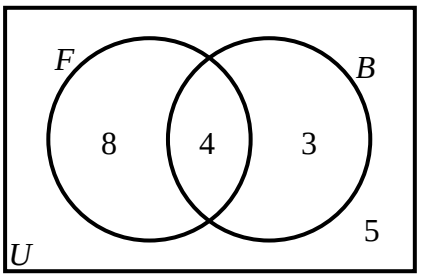
Mutual Exclusivity, Venn Diagrams and Probability

Level 1 – 2

1. For a class of 20 students, the Venn diagram on the right shows how many students play (*F*)ootball and (*B*)asketball.



For the following expressions:

i) explain the meaning
ii) calculate the value

a) $n(F \cap B)$ i) ……

ii) ……

b) $n(F')$ i) ……

ii) ……

c) $P(F \cup B)$ i) ……

ii) ……

d) Calculate the probability that a randomly selected person plays only one sport.

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e) Calculate the probability that a randomly selected person does not play either sport.

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f) Calculate the probability that a randomly selected person does not play both sports.

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g) Calculate the probability that a randomly chosen football player plays only football.

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2. Determine whether the following pairs of events are mutually exclusive. If they are not mutually exclusive give a reason.

a) A regular 6 sided die is rolled. Events A and B are:

A: A composite number is rolled

B: An odd number is rolled

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b) A card is randomly chosen from a standard 52-card deck. Events A and B are:

A: A red card is chosen

B: A spade is chosen

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c) Two coins are flipped. Events A and B are:

A: There is at least one tail

B: There are no heads

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d) In a lottery a number X is randomly chosen from the numbers 1 to 49. Events A and B are:

A: $X > 23$

B: $X \leq 23$

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e) A DP student is randomly chosen. Events A and B are:

A: The student studies mathematics HL

B: The student studies mathematics SL

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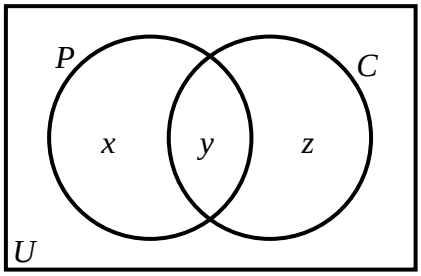
Level 3 – 4

3. In a class of 40 DP students everyone studies (*P*)hysics, (*C*)hemistry or both. The number of students who study physics is 28 and the number of students who study chemistry is 25.

   a) Use the Venn diagram to complete the equations on the right.



$x + y + z =$ .....

$x + y =$ .....

$y + z =$ .....

   b) Solve the equations to find the value of x, y and z.

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   c) Calculate the probability that a randomly selected student studies physics.

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   d) Calculate the probability that a randomly selected student studies physics, given that he/she studies chemistry.

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4. In a class of 30 students, all students study Mathematics or Physics. If 28 students study Mathematics and 15 students study Physics, with the help of a diagram find the number of students who:

   a) study only Mathematics ……

   b) study both Mathematics and Physics ……

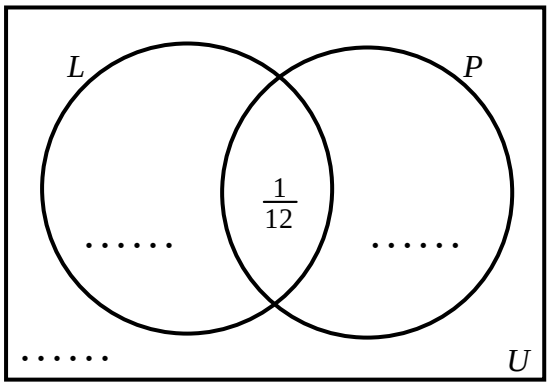
   c) study only Physics ……

Draw your diagram in this box

Use this box for working out

5. In a group of people, the probability that a person has visited London (L) is $\frac{7}{12}$. The probability that a person has visited Paris (P) is $\frac{4}{12}$. The probability that a person has visited both London and Paris is $\frac{1}{12}$.

   a) Use this information to complete the following Venn diagram:



   b) Calculate the probability a randomly selected person has visited:

   i) Only Paris ……

   ii) London but not Paris ……

   iii) Neither London nor Paris ……

   c) If these probabilities were calculated by surveying 120 people, how many people would you expect to have visited both cities?

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6. In a group of students the probability of choosing one who studies French is $\frac{1}{2}$. The probability of choosing one who studies Spanish is $\frac{2}{3}$. Are the two events disjoint? Explain your answer.

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7. Given that $P(A) = 0.3$, $P(B) = 0.5$ and $P((A \cup B)') = 0.2$ determine whether events A and B are mutually exclusive.

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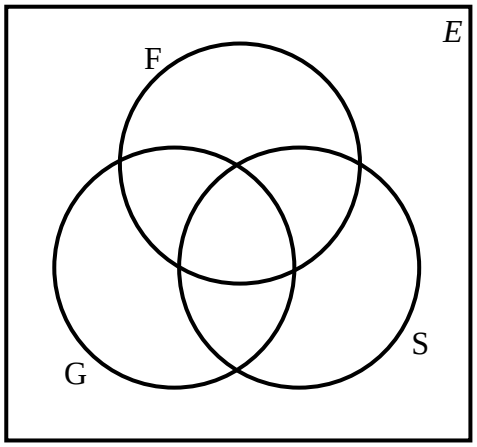
Level 5 – 6

8. In a class of 20 students everyone speaks English and some students speak other languages.

- 7 students speak French
- 10 students speak Spanish
- 9 students speak German
- 1 student speaks all three of these languages
- 3 students speak neither of these languages
- 2 students speak only German
- 3 students speak only French
- 4 students speak only Spanish



a) Complete the Venn diagram, showing how many students belong to each region.

b) A student is selected at random, calculate the probability of choosing a student who speaks:

i) only Spanish and French ……

ii) exactly two of the languages ……

iii) at least one of the languages ……

9. Sets *A*, *B* and *C* are defined as follows:

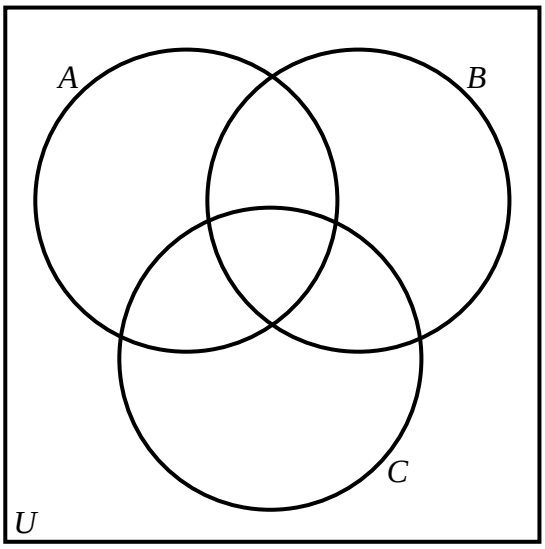
$A = \{\text{multiples of } 2\}$ $B = \{\text{prime numbers}\}$

$C = \{\text{numbers less than } 6\}$

a) Place the following numbers in the correct place on the Venn diagram:

2 3 6 7 9 10

11 13 15 19 20 21



b) A number is selected at random. Calculate the probability that the number belongs to the following sets:

i) $C \cap A$ ……

ii) $A \cup B'$ ……

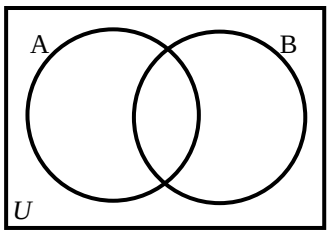
iii) $A \cup B \cup C$ ……

iv) $A \cup (B \cap C)$ ……

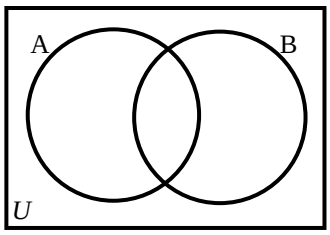
10. Create a Venn diagram showing the relationship between squares (S), quadrilaterals (Q), rectangles (R) and parallelograms (P).

Draw your diagram in this box

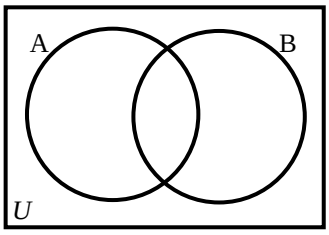
11. In each separate Venn diagram, shade the region indicated:

a) 

$A \cup B'$

b) 

$(A' \cap B) \cup (B' \cap A)$

c) 

$(A \cup B)' \cup (A \cap B)$

12. Let $P(A' \cap B) = 0.2$, $P(B) = 0.5$ and $P(A) = 0.4$

a) Determine whether events A and B are mutually exclusive

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b) Hence determine the value of $P(A \cap B)$.

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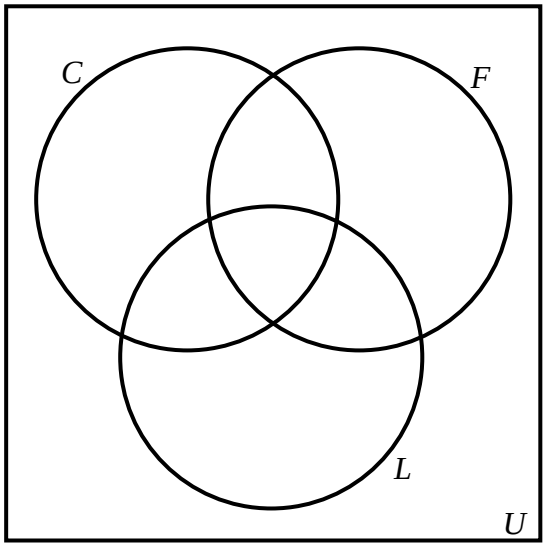
Level 7 – 8

13. A group of 40 students were surveyed to find which food they liked from a choice of chicken (C), fish (F) and lamb (L).

- 15 students like chicken
- 20 students like fish
- 8 students like fish and lamb, but not chicken
- 2 students do not like any of these foods
- 3 students like all three
- 9 students like chicken and fish
- 26 students like lamb

a) Represent this information on the following Venn diagram



b) How many students liked only chicken? ……

c) How many students liked only lamb? ……

d) How many students liked chicken and lamb but not fish? ……

e) A student is chosen at random. Calculate the following:

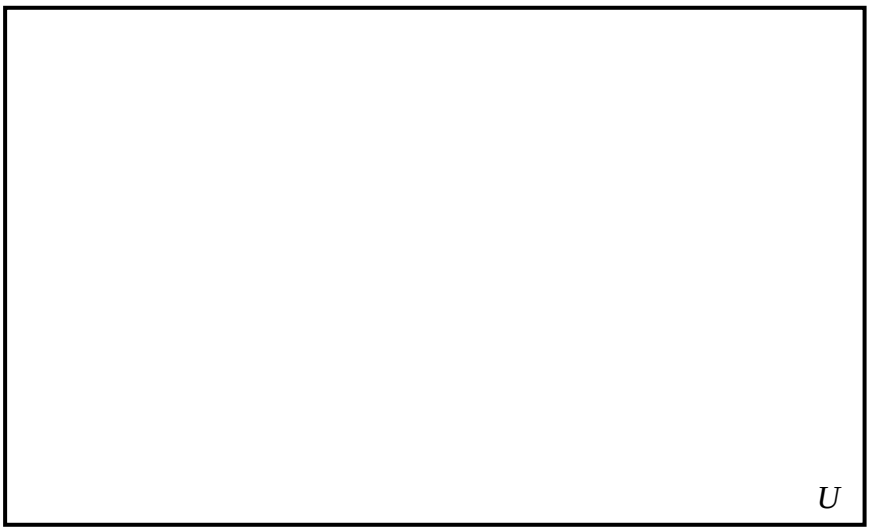
i) $P(C \cap F)$ …… ii) $P(L \cup C)$ ……

iii) $P(C \cap F \cap L)$ …… iv) $P(F \cap C')$ ……

v) $P(C \cup F \cup L)'$ …… vi) $P\left[(F \cap L) \cup C\right]$ ……

14. Let U represent the pets in a pet shop. Some (D)ogs are (B)rown. Some (C)ats are brown. No (R)abbits are brown.

a) Represent the sets D, B, C and R in a Venn diagram.



b) Add the following pet IDs to your diagram (do not add any more regions to your diagram):

ID	Name	Type	Colour
1	Felix	Cat	White
2	Garfield	Cat	Brown
3	Snoopy	Dog	Brown
4	Richard	Hamster	White
5	Scooby	Dog	Black
6	Peter	Rabbit	Gray
7	Tom	Cat	White
8	Pluto	Dog	Black
9	Roger	Rabbit	Gray
10	Jerry	Mouse	Brown

c) Calculate the probability a randomly chosen pet form the list above belongs to the following sets:

i) $D \cup (B \cap C)'$ …….

ii) $R \cup B$ …….

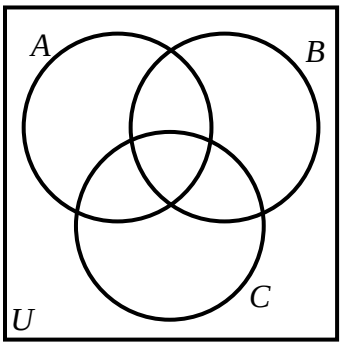
iii) $B' \cup R'$ …….

d) Write, using mathematical notation, the sets which contain *only* the following animals:

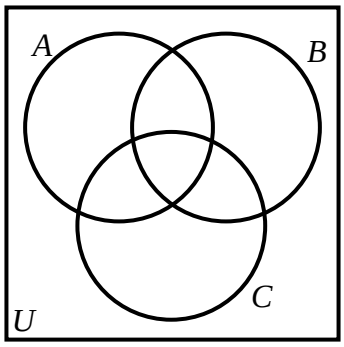
i) Jerry, Tom and Felix …….

ii) Snoopy and Richard …….

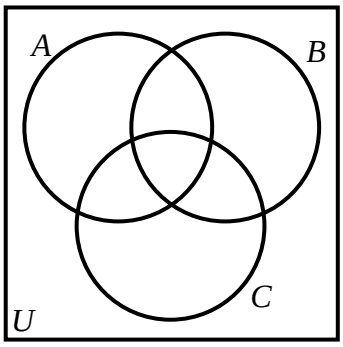
15. In each separate Venn diagram, shade the region indicated:

a) 

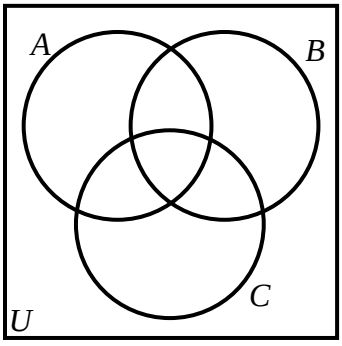
$A \cup (B \cap C')$

b) 

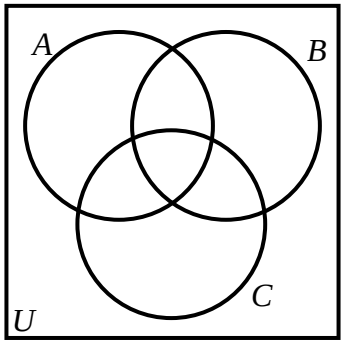
$B \cap (A \cup B)'$

c) 

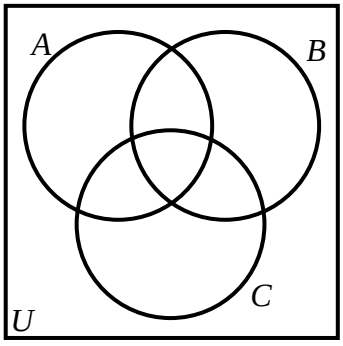
$A \cap B \cap C'$

d) 

$(A \cup B) \cap C'$

e) 

$(A \cup B \cup C)'$

f) 

$A' \cap B' \cap C'$