

Infinite Geometric Series

Level 1 – 2

1. Which of the following infinite geometric series have a finite value? Circle your choices.

a) $4 + 8 + 16 + 32 + \dots$

b) $100 + 50 + 25 + 12.5 + \dots$

c) $400 - 40 + 4 - 0.4 + \dots$

d) $7 + 7 + 7 + 7 + \dots$

e) $1 - 3 + 9 - 27 + \dots$

f) $1 + 0.1 + 0.01 + 0.001 + \dots$

2. Calculate the value of the following infinite geometric series:

a) $t_1 = 10, r = \frac{1}{2}$

b) $t_1 = 20, r = \frac{1}{4}$

c) $t_1 = 1, r = -\frac{1}{2}$

d) $t_1 = -5, r = \frac{2}{3}$

e) $t_1 = -100, r = -\frac{4}{5}$

f) $t_1 = 4, r = \frac{1}{10}$

3. Calculate the value of t_1 for the following infinite geometric series:

a) $S_\infty = 2, r = \frac{1}{2}$

b) $S_\infty = -3, r = \frac{1}{4}$

c) $S_\infty = 20, r = -\frac{1}{2}$

d) $S_\infty = 100, r = \frac{2}{5}$

e) $S_\infty = -10, r = -\frac{2}{3}$

f) $S_\infty = 3.\dot{3}, r = \frac{1}{10}$

Level 3 – 4

4. If it exists, calculate the value of each infinite geometric series.

a) $100 + 30 + 9 + 2.7 + \dots$

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b) $3 + 6 + 12 + 24 + \dots$

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c) $20 - 2 + 0.2 - 0.02 + \dots$

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d) $900 + 810 + 729 + 656.1 + \dots$

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e) $-30 + 3 - 0.3 + 0.03 - \dots$

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f) $2 - 6 + 18 - 54 + \dots$

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5. Calculate the value of r for the following infinite geometric series:

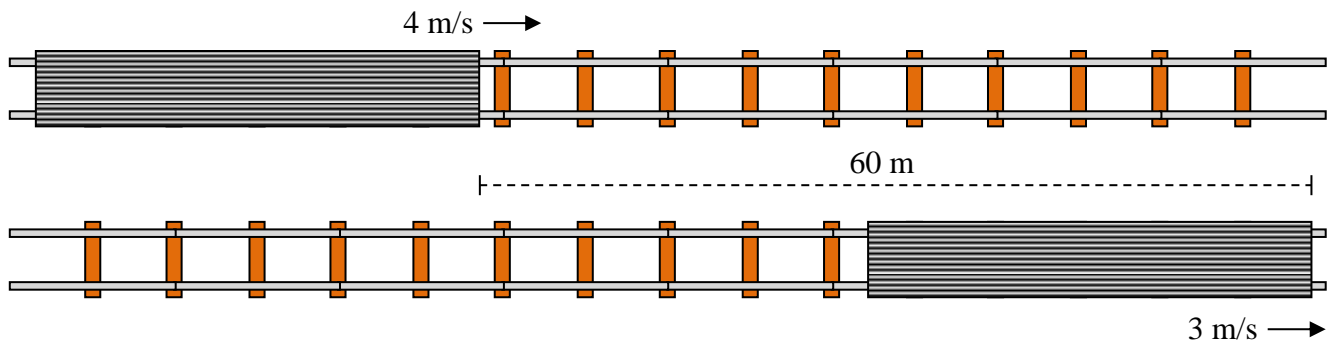
a) $S_{\infty} = 4, t_1 = 5$

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b) $S_{\infty} = 10, t_1 = 5$

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7. Two trains are travelling along parallel tracks. Train A is travelling at 4 m/s and train B is travelling at 3 m/s. Initially, train B is 60 m ahead of train A.



a) i) Calculate how long it will take train A to reach train B's current position.

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ii) During this time how far will train B travel?

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b) Repeat the steps in part a) using the new positions of the trains.

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i)

ii)

c) i) Write down the initial distance between the trains

ii) Write down the distance between the trains after part a)

iii) Write down the distance between the trains after part b)

iv) Repeat the same steps two more times and write down the distance between the trains at the end of each step.

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v) What kind of sequence has been generated? Arithmetic / Geometric

d) i) Write down an expression for the total distance travelled by train A after n steps.

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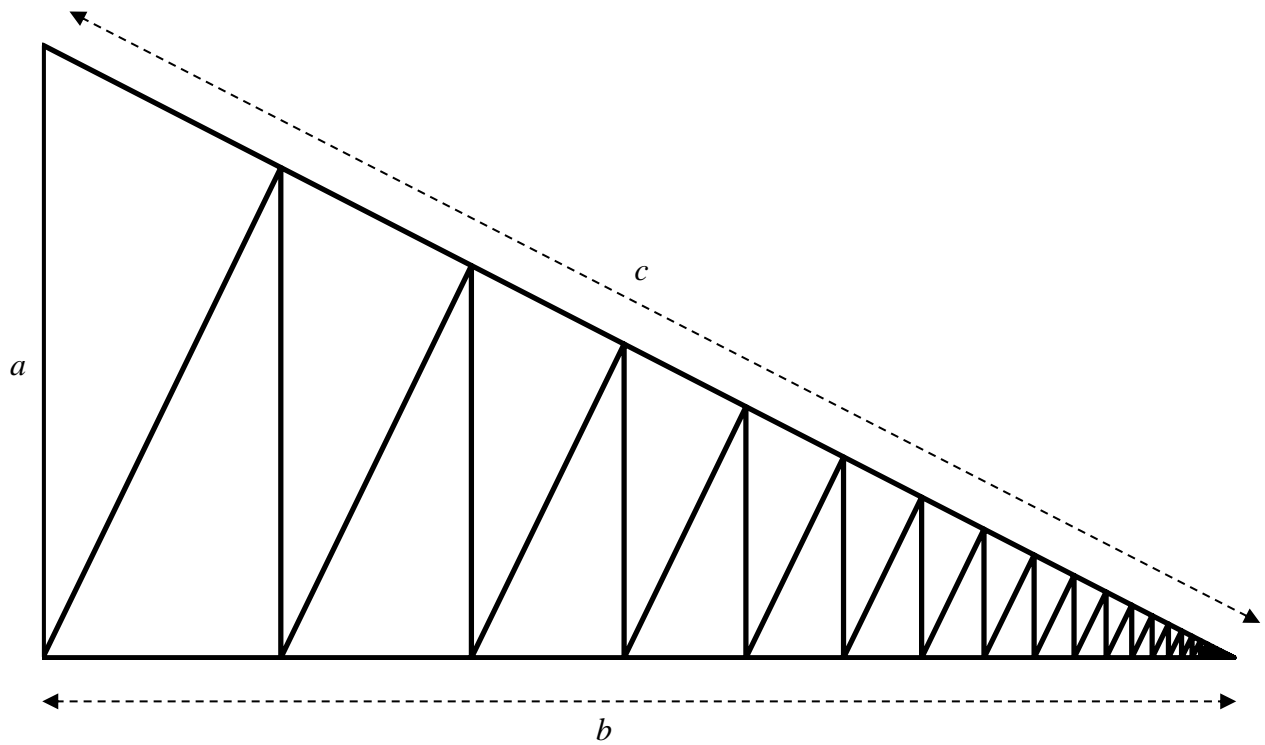
ii) As the number of steps increases to infinity, find the total distance travelled by train A.

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iii) What is the significance of your answer to part ii)?

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8. Consider the following right-angled triangle, which is divided into infinitely many smaller (and similar) right-angled triangles. (*The aim of this question is to prove the Pythagorean relationship, therefore you cannot use this relationship in any part of the question*).



- a) Find the areas of the first three triangles from the left, in terms of a , b and c .

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- b) Show that these three areas form a geometric sequence.

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c) Write down the area of the original large triangle in terms of a and b .

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d) By considering the infinite geometric series formed by the area of the small triangles, find an expression for the total area, in terms of a , b and c .

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e) Hence prove the Pythagorean relationship $a^2 + b^2 = c^2$.

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9. The sum to infinity of an infinite geometric series is 4. The sum of the first three terms is 2.75. Determine the first term and the common ratio.

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10. Paul and Steven play a game where they each take turns to roll a die. The winner is the first person to roll a 6. Paul rolls first.

a) Determine the probability that Paul will win on his

i) first roll

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ii) second roll

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iii) third roll

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b) Determine the probability that Paul will win the game.

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c) Mike joins the game. If Paul rolls first, then Steven then Mike, determine the probability of each person winning.

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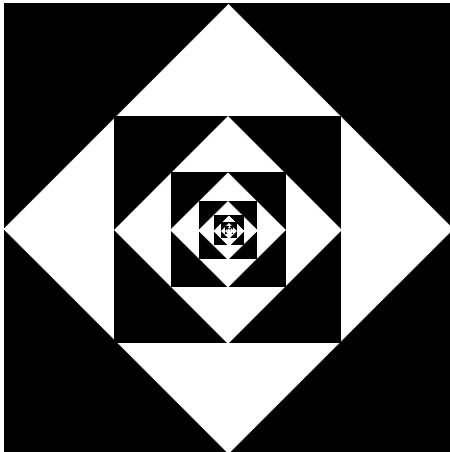
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12. The length of one side of the outside square is 10 cm. Every smaller square is formed by joining the midpoints of its parent square. If the pattern continues to infinity determine the total area of the black region.



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13. Paul, Steven and Mike play a game of paintball, taking it in turns to take one shot. If a person gets hit he is out of the game. Paul hits his target 30% of the time, Steven hits his target 50% of the time and Mike hits his target 100% of the time. Paul goes first, then Steven and then Mike. They continue to shoot in this order until there is a winner (once a person is hit he no longer takes part in the game). If each person knows how skillful everyone else is, and each person chooses the best strategy each time he shoots, determine the probability of each person winning. A player may make an intentional miss if it will increase his chances of winning.

Use a separate sheet of paper if you wish to attempt this question. A question like this will never appear on a test. The result may be surprising.