Binomial Expansion

Level 1 – 2

To determine a number in Pascal's triangle we simply calculate the sum of the two numbers above.

1. Complete the missing values:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

↘ + ↙ ↘ + ↙

1 5 10 10 5 1

1 6 …… …… …… …… …… 1

1 …… …… …… 35 …… …… …… 1

2. We can use a function to express any number in Pascal's triangle. The r^{th} number in the n^{th} row is written

$$^{n}C_{r} \quad \text{or} \quad \binom{n}{r}$$

Note that we start counting both *n* and *r* from zero. For example:

$$\binom{4}{2} = 6 \qquad {}^{5}C_{4} = 5 \qquad \binom{3}{0} = 1 \qquad {}^{3}C_{2} = 3$$

a) Use the previous question to write down the following values:

i) $^{7}C_{2}$ …………………

ii) $\binom{6}{4}$ …………………

iii) $^{3}C_{0}$ …………………

iv) $\binom{5}{5}$ …………………

We can use a calculator to determine any number in Pascal's triangle.

Example: Use a TI-84 to determine $^{8}C_{3}$.

Press 8 MATH and select nCr from the PRB menu and then press 3 ENTER.

All other scientific calculators also have an nCr function.

Use your calculator to determine:

a) $^{10}C_{2}$ ………………… b) $\binom{12}{5}$ …………………

c) $\binom{15}{3}$ ………………… d) $^{9}C_{7}$ …………………

Level 3 – 4

3. There is also an equation we can use to calculate the value of $^{n}C_{r}$:

$$^{n}C_{r}=\frac{n!}{r!(n-r)!} \qquad \text{where } n!=n\times(n-1)\times(n-2)\times\ldots\times2\times1$$

Example

$$^{7}C_{2}=\frac{7!}{2!(7-2)!}$$

$$=\frac{\overbrace{7\times6\times5\times4\times3\times2\times1}^{7!}}{\underbrace{2\times1}_{2!}\times\underbrace{5\times4\times3\times2\times1}_{(7-2)!=5!}}$$

$$=\frac{7\times6\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{2\times1\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}$$

$$=\frac{42}{2}=21$$

By hand, determine the following. Show full working out:

a) $^{6}C_{4}$ ………………………………………………………………………………

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b) $\begin{pmatrix} 8 \\ 2 \end{pmatrix}$ ……………………………………………………………………………

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c) ${}^{9}C_{7}$ ……………………………………………………………………………

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4. Expand and simplify the following. The first two have been completed. The third has been *started*. **Show full working out**.

a) $(x+1)^2$ $(x+1)(x+1) = x^2 + x + x + 1$

$= x^2 + 2x + 1$

b) $(x+1)^3$ $(x+1)(x+1)^2 = (x+1)(x^2 + 2x + 1)$

$= x^3 + x^2 + 2x^2 + 2x + x + 1 = x^3 + 3x^2 + 3x + 1$

c) $(x+1)^4$ $(x+1)(x+1)^3 =$

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d) $(x+1)^5$ ……………………………………………………………………………

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Level 5 – 6

5. Look at your answers to the previous question and describe any patterns you see. *Hint: compare your answers to the numbers in Pascal's triangle.*

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6. Determine the coefficient of x^3 (= the number in front of x^3) in the expansion of $(x+1)^7$.

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7. Determine the coefficient of x^6 in the expansion of $(x+1)^9$.

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8. Determine the coefficient of x^{10} in the expansion of $(x+1)^{16}$.

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9. Determine the coefficient of x^3 in the expansion of $(x+1)^{20}$.

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10. Determine the coefficient of x^5 in the expansion of $(x+1)^{22}$.

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Level 7 – 8

11. Expand and simplify the following:

a) $(a+b)^2$ ………………………………………………………………………

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b) $(a+b)^3$ ………………………………………………………………………

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c) $(a+b)^4$ ………………………………………………………………………

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d) $(a+b)^5$ ………………………………………………………………………

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12. Use your results or any patterns you notice in the previous question to determine the coefficient of

a) x^3 in the expansion of $(2x+3)^5$ ………………………………………………………………

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b) x in the expansion of $(2-x)^4$ ………………………………………………………………

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c) x^5 in the expansion of $(-2x+5)^7$ ………………………………………………………………

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