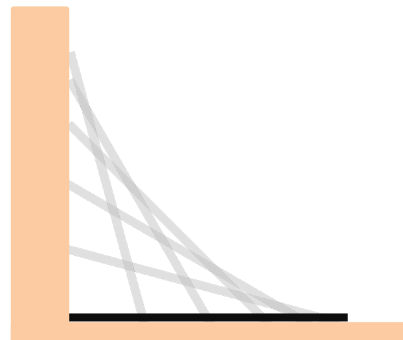


Geogebra Tutorial

Consider the following problem:

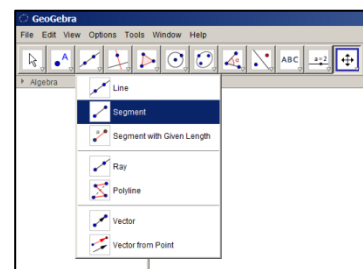
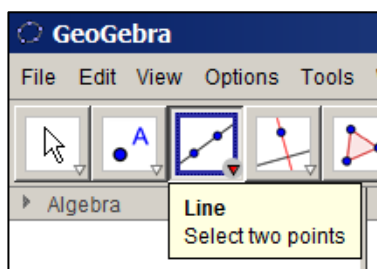
A 10 m long ladder is leaning against a wall. A man is standing exactly half way up the ladder. The ladder slips so that the top of the ladder slides down the wall, and the bottom of the ladder slides along the floor, eventually landing horizontally (shown in the diagram on the right). If the floor and the wall represent the x and y -axis respectively, determine the equation of the path that the man travels.



Drawing Line Segments

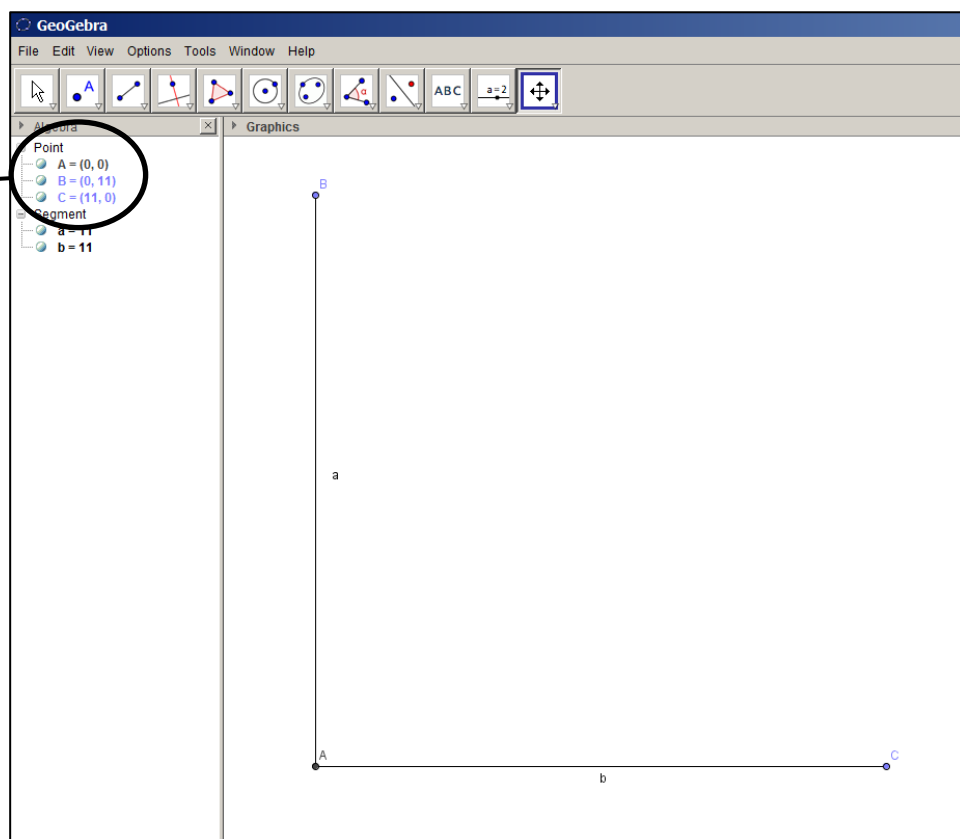
Select the segment tool. If you cannot see it, click on the white arrow on the third button to reveal more functions.

Draw a segment from $(0,0)$ to $(0,11)$. This will represent the wall. Draw a segment from $(0,0)$ to $(11,0)$. This will represent the floor.



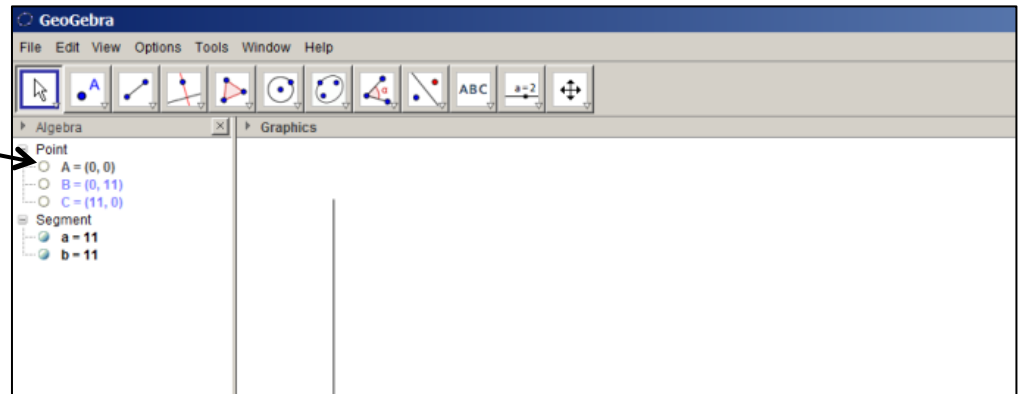
Right click on the screen and turn off the *Axes*:

If you didn't create your points accurately you can type the exact values that you want here.



The two lines are labelled with the letters a and b . If we right-click on each line you can hide these labels. It is not necessary to display points A, B and C. If we click the green dots next to each point in the *Algebra* box it will hide the points:

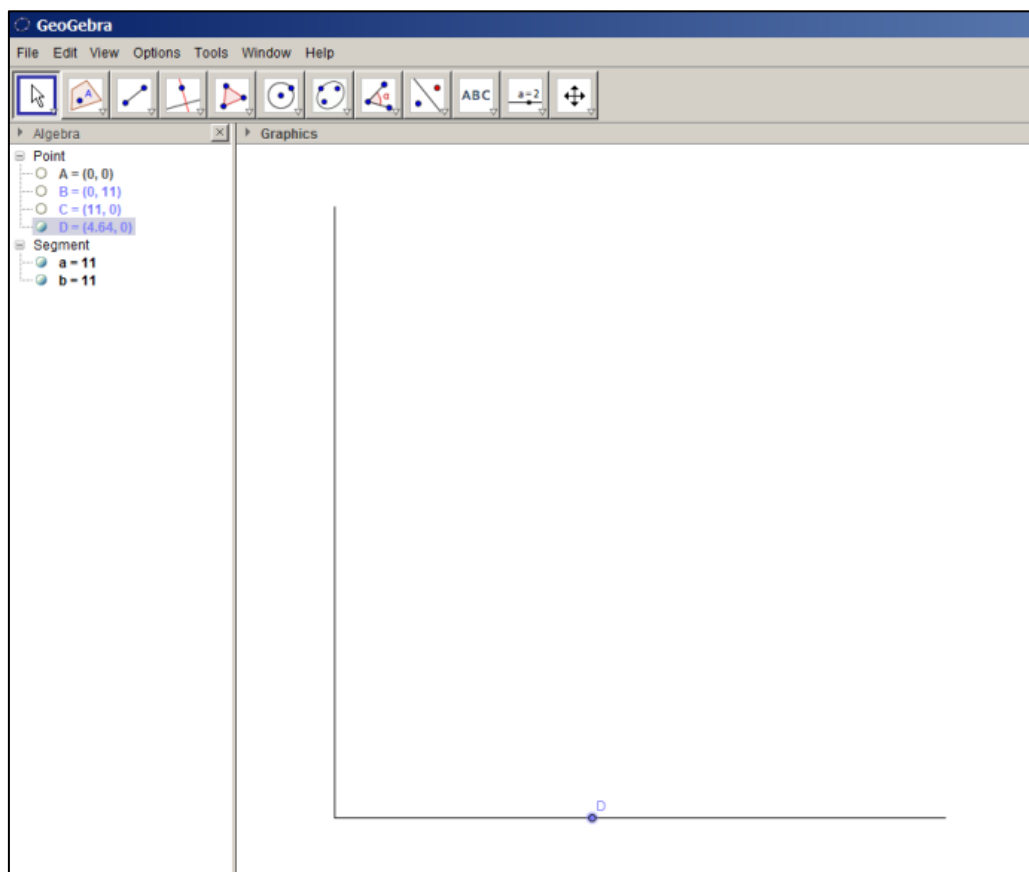
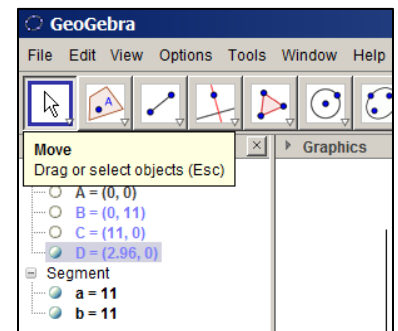
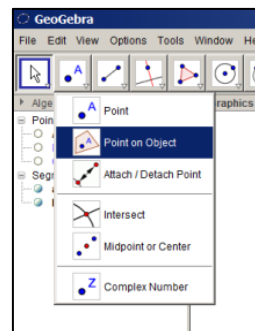
Hide the points by clicking on the green dots.



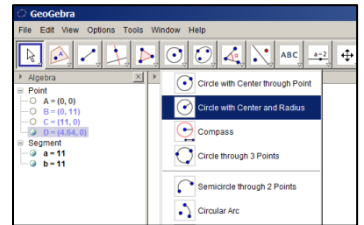
Restricting a Point to a Line

The base of the ladder is restricted to the floor, so we need to create a point, which represents the base of the ladder, that is restricted to the horizontal line. Select the *Point on Object* tool and place a point on the horizontal line.

If we try and move the point, using the *Move* tool we will see that it is restricted to the floor.

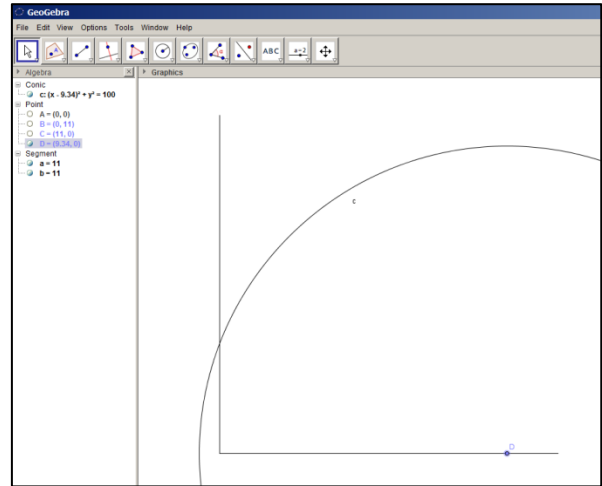
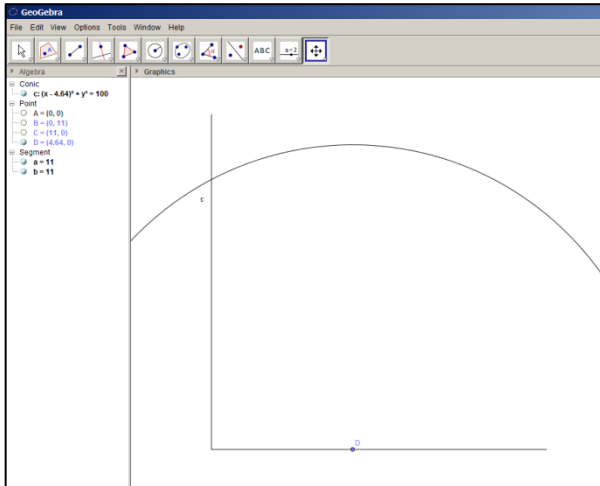


The top of the ladder is restricted to the wall. However, we cannot simply create another random point on the wall. The ladder is 10 m in length so the point on the wall must always be at a distance of 10 from the point on the floor.



Drawing Circles

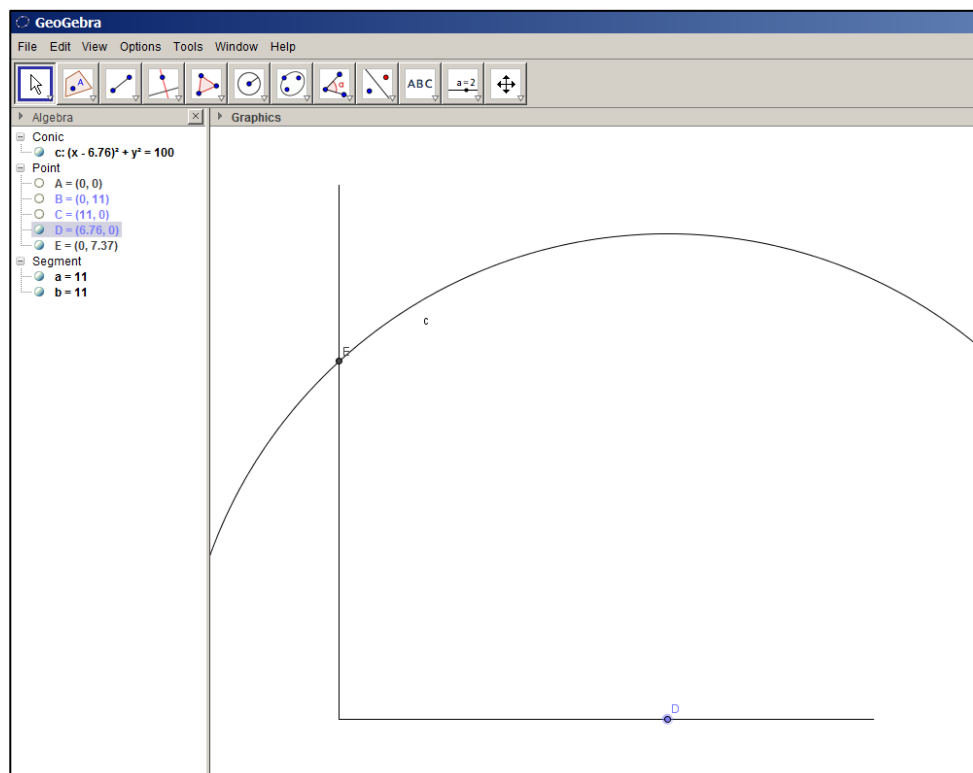
Select the *Circle with Centre and Radius* tool (note that there are several tools to create a circle). Draw a circle centred at point D with radius 10. Now, try moving point D. Notice that the circle will always move with it.



The point where the circle intersects the vertical line represents the end of the ladder (it will always be 10 m from point D).

Select the *Point on Object* tool and create a new point at the intersection of the circle and the wall.

Test to see if it has been created correctly by moving point D. Our new point should always move to where the circle intersects the wall.

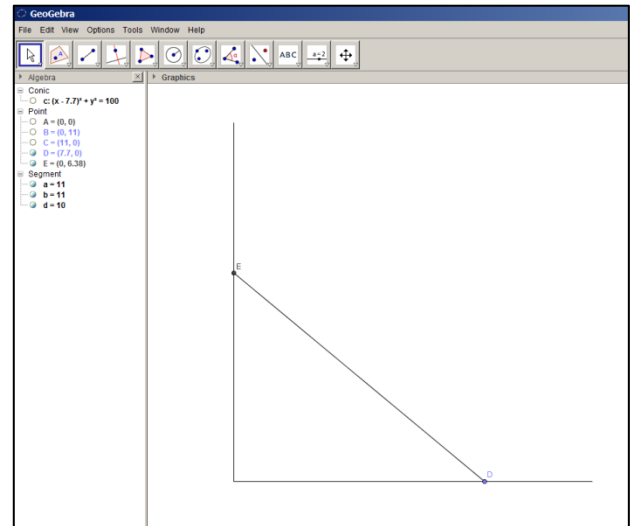


Now we can hide the circle. A quick way to do this is by clicking the green dot next to the equation of the circle in the *Conic* section of the *Algebra* window.

Draw a line segment from point D to point E. This will represent the ladder. Move point D away from the wall and the ladder should move (slip) down the wall.

The ladder is labelled with a letter d . This can be hidden by right-clicking on the ladder and unchecking *Show Label*.

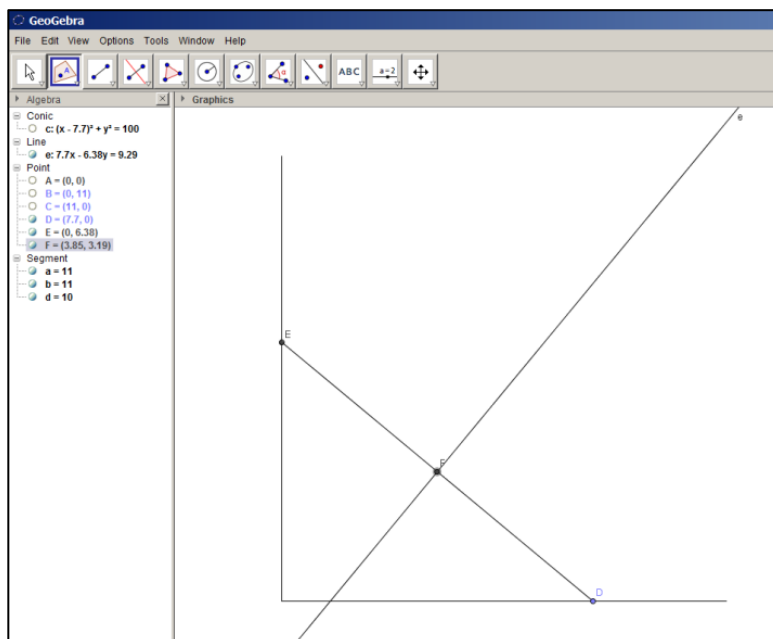
We now need to create a point that represents the man standing on the ladder, half way along its length.



Perpendicular Bisectors

Locate and select the *Perpendicular Bisector* tool and click on the ladder.

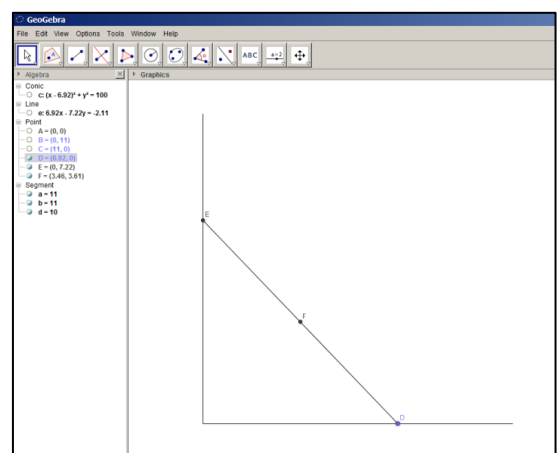
Select the *Point on Object* tool and create a new point where the perpendicular bisector intersects the ladder:



We can hide the perpendicular bisector and the model is complete.

Move point D to see the path of the man as the ladder slips.

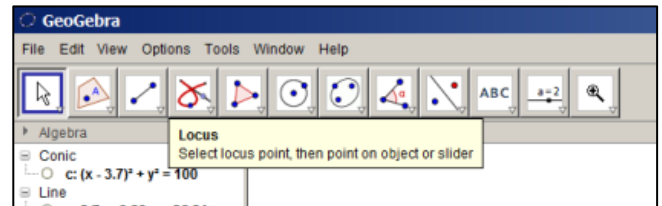
We now need to use the model to determine the equation of the path of the man.



Loci

We can use Geogebra to draw the path of the man as the ladder slides down the wall.

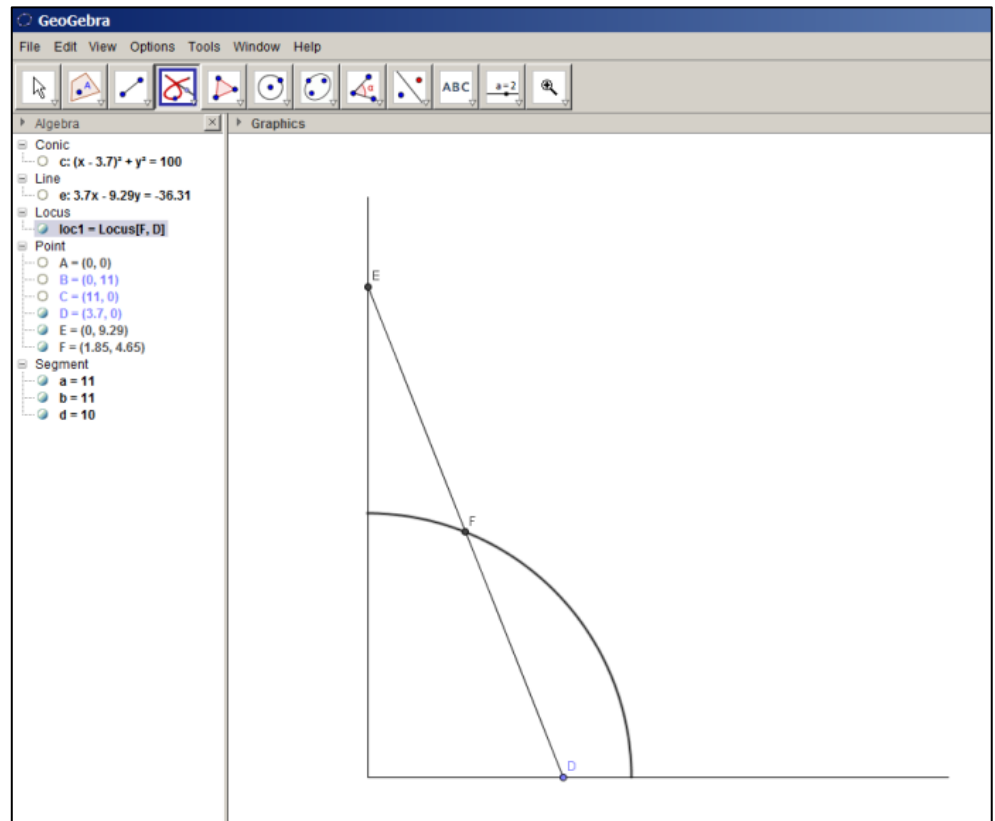
Locate and select the *Locus* tool. Notice that if you hold the mouse over the tool it will give you a description of how to use it.



Click on the man (point F) and then click on the base of the ladder (point D). The path of the man will then be drawn onto the screen.

The path appears to be a quarter of a circle, centred at the origin.

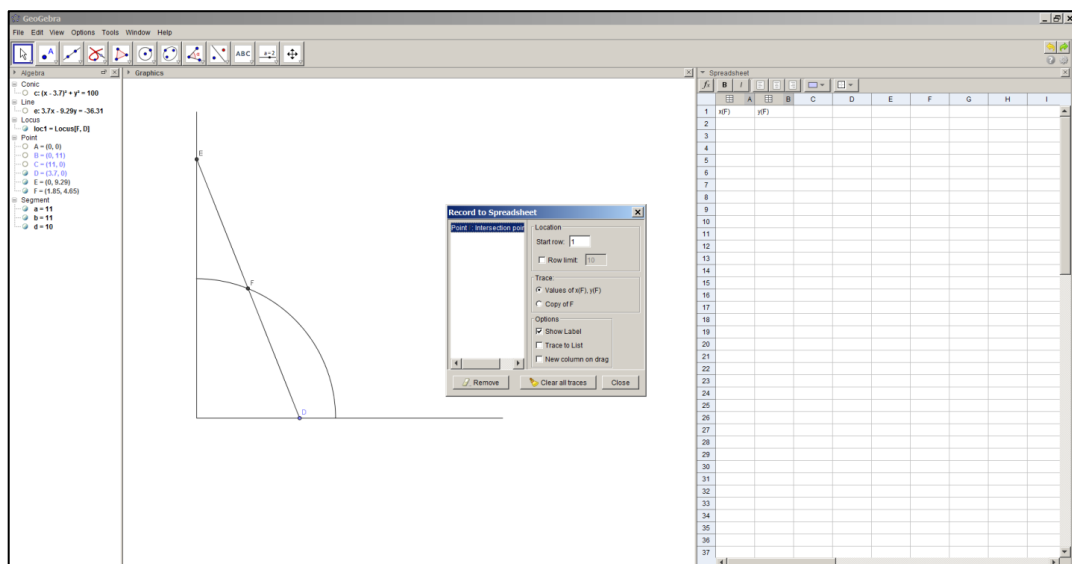
There is another method we can use to confirm our theory that the path of the man is circular. Geogebra contains a built in spreadsheet that can record the coordinates of a point.



Recording to the Spreadsheet

Select *View > Spreadsheet* and the spreadsheet will appear on the right of the screen.

Right-click on point F and select *Record to Spreadsheet*. Then click *Close* to close the pop-up window.



Now, move point D and its coordinates will be recorded to the spreadsheet.

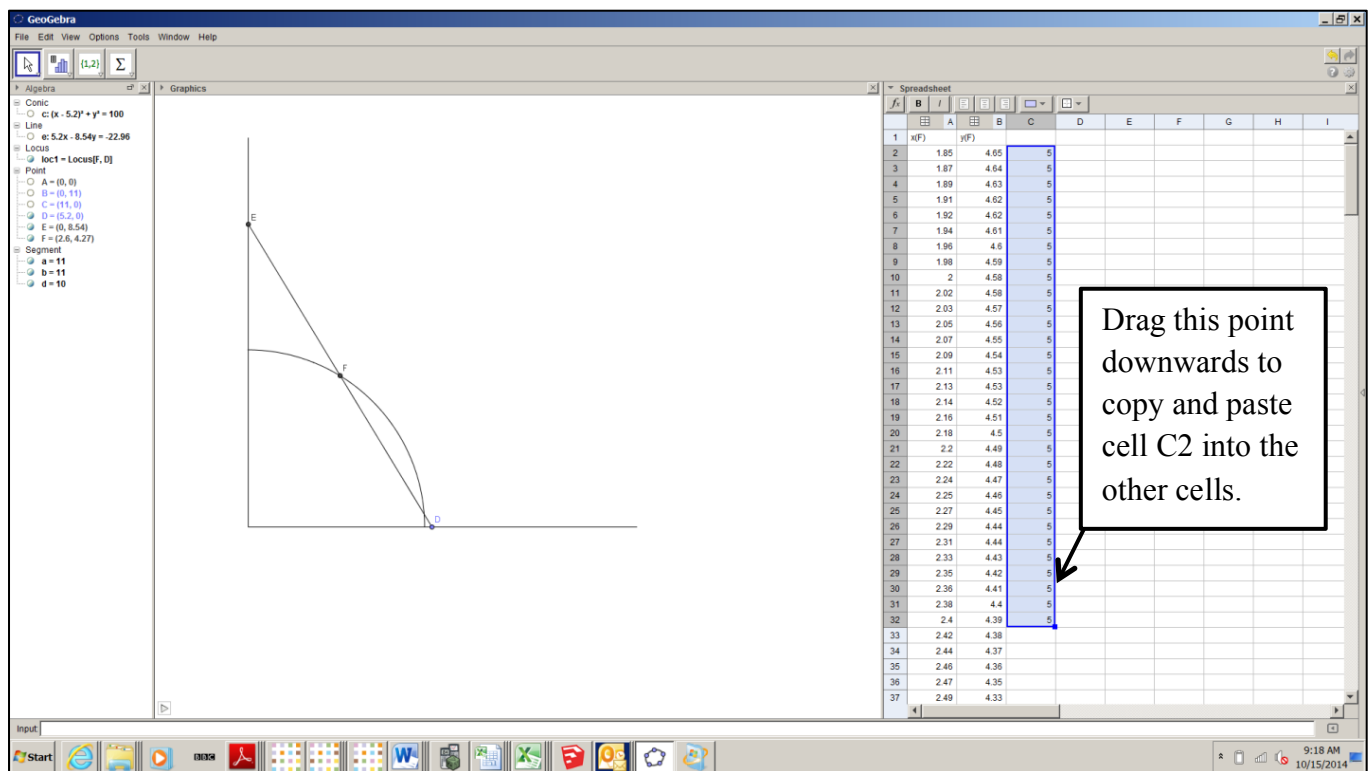
Alternatively, right-click point D and select *Animation On* and it will automatically move. To pause the animation press the pause button in the bottom left corner.

Our spreadsheet will now contain many coordinates representing the various positions of F. If the path of F is indeed circular, centred at the origin, then it will always be a fixed distance from the origin. We can check this by using the Pythagorean theorem.

In cell C2 type: $\text{=sqrt}(A2^2+B2^2)$

This calculates the value of $\sqrt{x^2 + y^2}$, which is the distance from the origin.

If we copy and paste this formula into all of the other rows by dragging the bottom right corner of the cell downwards, we can see that the distance of point D from the origin is always equal to 5.



The position of the man can be given by the equation:

$$y = \sqrt{25 - x^2}$$

Note that we haven't actually proven that this equation is true; although the evidence given by Geogebra strongly hints that it is true. To prove it we have to use algebra.

Proof:

The coordinates of the base of the ladder are $(t, 0)$ for some $t \in [0, 10]$.

The coordinates of the top of the ladder are therefore $(0, \sqrt{100 - t^2})$.

The coordinates of the man are therefore $\left(\frac{t}{2}, \frac{\sqrt{100 - t^2}}{2}\right)$.

So we have:

$$x = \frac{t}{2} \Rightarrow t = 2x$$

$$\begin{aligned} y &= \frac{\sqrt{100 - t^2}}{2} \\ &= \frac{\sqrt{100 - 4x^2}}{2} \\ &= \sqrt{25 - x^2} \end{aligned}$$

This completes the proof.