How Do Simple Rules Create Complicated Structures?

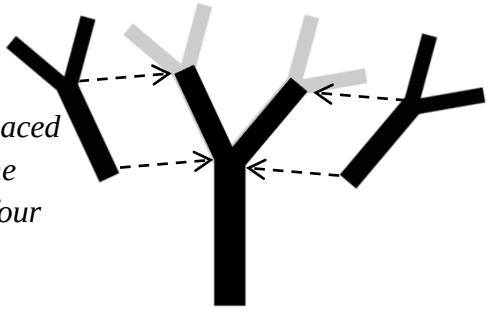
A fractal tree is created by repeatedly replacing each branch with a smaller version of the original design.

Step 1:

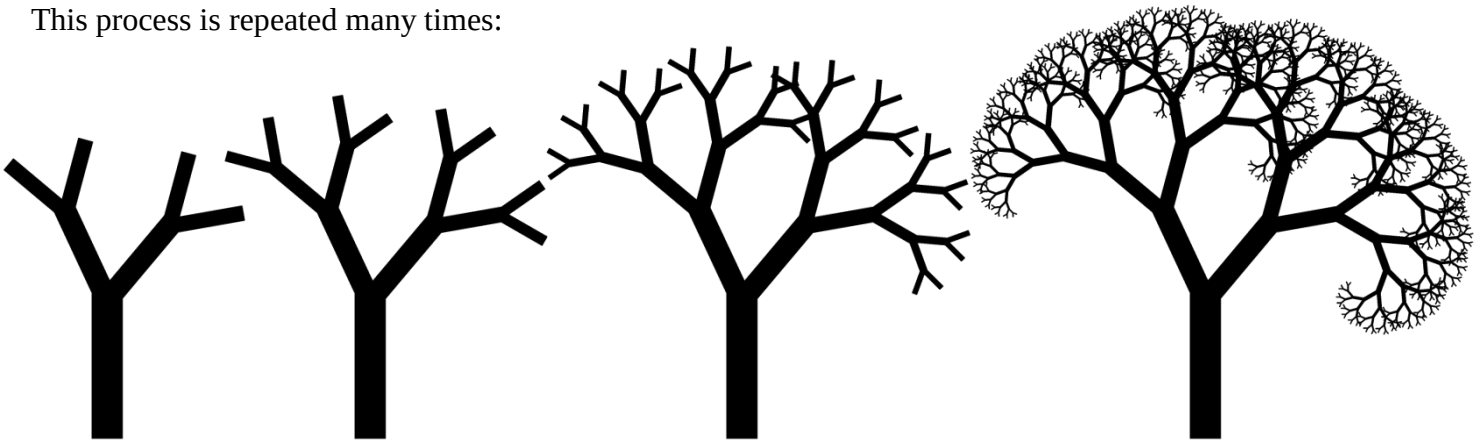
This is the original design. The two branches are 70% of the length and width of the trunk.

Step 2:

The two branches are replaced with smaller versions of the original design, creating four new branches.



This process is repeated many times:



Step 2 Step 3 Step 5 Step 10

In each step, the length of each new branch is 70% of the length of its parent branch.

Task 1 (This corresponds to levels 1 – 4 of criterion B, along with all levels of criterion C)

- Decide a length for your trunk and a percentage which you will use to calculate the lengths of your branches. Each branch should be the same length. For example, if you choose 20 m and 75% then your trunk will be 20 m and your two branches will be 75% of 20 = 15 m.

- Calculate i) the total length of each individual new branch during the first five steps and ii) the total length of all new branches at each step for the first five steps.

- Add one more row to your table. The step number should be equal to *n*. Fill in the rest of the row.

Use a table to help you organize your data. For example:

Step	Number of new branches	Length of each new branch	Total length of all new branches
1	2	20×0.75	40×0.75
2	4	20×0.75^2	80×0.75^2
etc.			

Leave your answers in index form as shown in the table. This will make it easier to see any patterns.

Task 2 (This corresponds to levels 5 – 6 of criterion B, along with all levels of criterion C)

- Create a new tree but give the two branches in the original design a different length (to each other). Calculate the total length of all new branches at each stage for the first five steps.

Task 3 (This corresponds to levels 7 – 8 of criterion B, along with all levels of criterion C)

- Suppose the length of the trunk is L, and the lengths of the two branches in step 1 are aL and bL respectively. Use your previous results to derive an expression for the total length of all new branches after n steps. Your method should be clearly explained and your expression should be completely factorized.
- *Extended students may wish to derive a formula for the total length of <u>all</u> branches and the trunk after n steps. For this, you will need to research the formula for the value of a geometric series. However, you will not be graded on this.*

END OF INVESTIGATION

Notes:

- *Different angles and different scales produce different trees; some are more beautiful than others. If you are not happy with your design, start again.*
- *Submit your investigation as a report written in Microsoft Word. Use Sketchup to create Fractal Tree diagrams. Export them as 2D images and import then into your Word document.*
- *Make sure you include an introduction in your report. Your introduction should include an explanation, **in your own words**, of how fractal trees are formed, along with step-by-step diagrams. It should also include a very detailed fractal tree, which includes far more than 5 steps.*
- *Plan your investigation before you start creating. Your report should include step by step diagrams. It is easier to create those diagrams while you are making your trees, not after you have made your trees.*
- *Your report should be able to be understood by anyone reading it, without having to refer to this instruction sheet.*
- *Pascal's Triangle may be useful for some parts of this investigation.*

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **use** limited mathematical language II. **use** limited forms of mathematical representation to present information III. **communicate** through lines of reasoning that are difficult to interpret.	The student is able to: o explain how fractal trees are created o create simple fractal trees using Sketchup
3 – 4	The student is able to: I. **use** some appropriate mathematical language II. **use** different forms of mathematical representation to present information adequately III. **communicate** through lines of reasoning that are able to be understood, although these are not always clear IV. adequately **organize** information using a logical structure.	The student is able to: o explain how fractal trees are created using explanations and diagrams o create fractal trees using Sketchup o use tables to arrange data
5 – 6	The student is able to: I. usually **use** appropriate mathematical language II. usually **use** different forms of mathematical representation to present information correctly III. move between different forms of mathematical representation with some success IV. **communicate** through lines of reasoning that are clear although not always coherent or complete V. present work that is usually **organized** using a logical structure.	The student is able to: o explain how fractal trees are created in his/her own words using explanations and step-by-step diagrams o create a detailed fractal tree using Sketchup, with many steps o hide lines used in the creation of fractal trees o use tables to arrange data o display formulae using the equation editor o move effectively between explanations, calculations, tables and diagrams
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use different forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is consistently organized using a logical structure.	The student is able to: o clearly explain how fractal trees are created in his/her own words using explanations and step-by-step diagrams o step-by-step diagrams of a growing fractal tree are all displayed to the same scale o create a detailed fractal tree using Sketchup, with many steps o hide all lines used in the construction of fractal trees o display data clearly and accurately in tables o display formulae clearly and accurately using the equation editor o move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 3 demonstrates, etc.) o make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) o create a report that is able to be understood without referring to the task sheet

Criterion B: Investigating Patterns		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **apply, with teacher support**, mathematical problem-solving techniques to discover **simple patterns** II. **state predictions** consistent with patterns.	The student is able to: ○ complete the tables for steps 1 to 5 in task 1.
3 – 4	The student is able to: I. **apply** mathematical problem-solving techniques to discover **simple patterns** II. **suggest general rules** consistent with **findings**.	The student is able to: ○ complete the tables for steps 1 to 5, and the n^{th} term, in task 1.
5 – 6	The student is able to: I. **select and apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **findings** III. **verify** the validity of these general rules.	The student is able to: ○ complete the tables for steps 1 to 5, and the n^{th} term, in task 1. ○ calculate the total length of all new branches for steps 1 to 5 in task 2.
7 – 8	The student is able to: I. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **correct findings** III. **prove, or verify and justify**, these general rules.	The student is able to: ○ complete the tables for steps 1 to 5, and the n^{th} term, in tasks 1. ○ calculate the total length of all new branches for steps 1 to 5 in task 2. ○ clearly and accurately derive an expression for the total length of all new branches after n steps, in task 3.