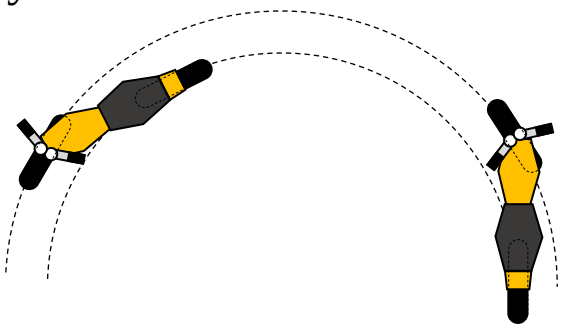
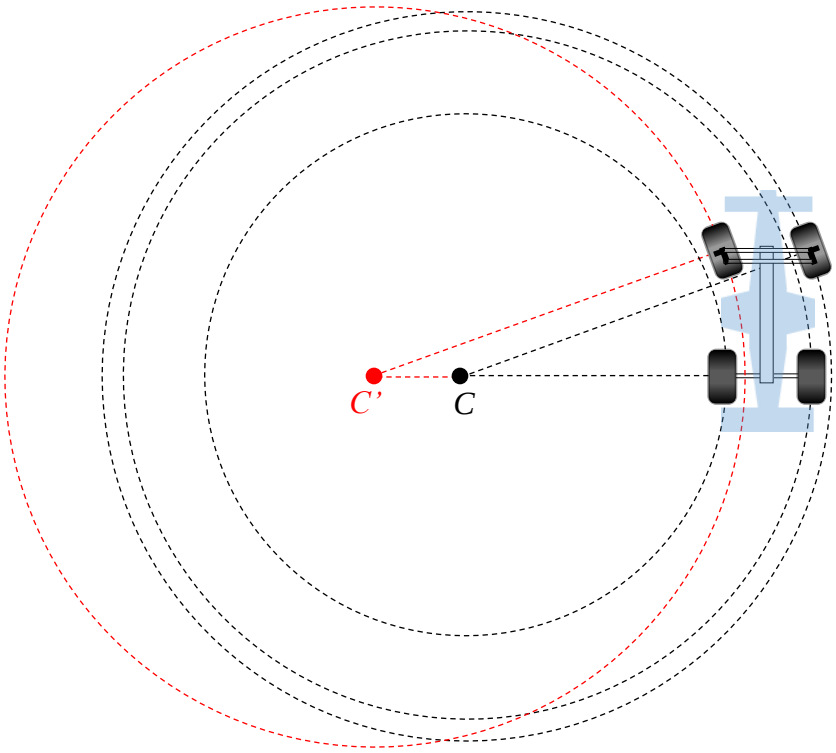
Designing a Steering System

When a motorbike turns a corner, each wheel follows a different circular arc. Each wheel is tangential to each arc at all times. This is shown in the diagram on the right.



The arcs both have the same centre, but their radii depend on the angle at which the front wheel is turned, and the distance between the centres of the wheels.



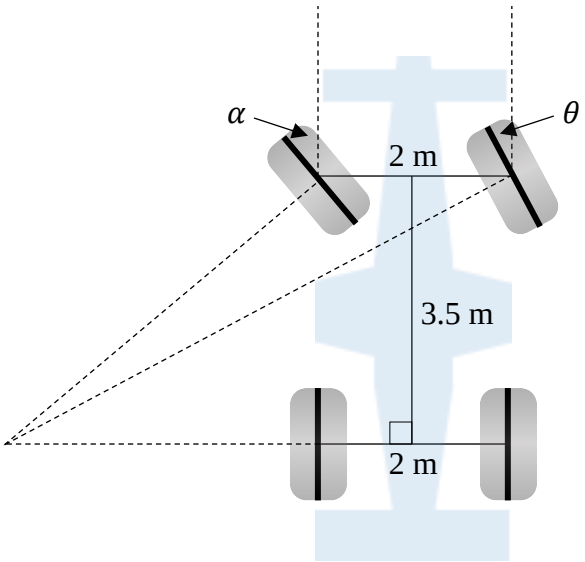
Suppose the front wheels of a car both turn at the same angle. When a car turns, its weight is pushed to the outside wheels, so it is the outside wheels which determine the direction in which the car travels. This results in the car travelling in an arc with a different centre to the arc in which the inside-front wheel wants to travel.

This is shown in the diagram on the left. The car will follow the black circles with centre *C*, but the inside-front wheel wants to follow the red circle with centre *C*'. This results in the inside-front wheel rubbing against the ground, and wearing out the tyre.

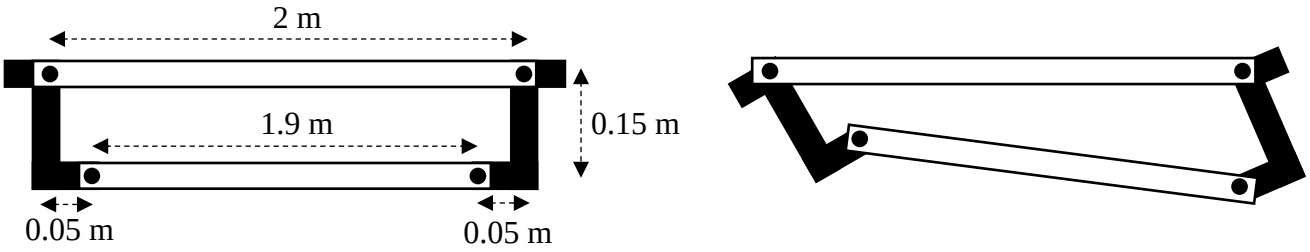
We need to develop a steering sytem so that both of the front wheels naturally follow arcs with the same centre.

Before we can develop a system, we should determine at which angles the inside-front wheel should turn when the outside-front wheel has turned by specific angles. Suppose the distance between the centres of the front and rear wheels is 3.5 m and the distance between the centres of the left and right wheels is 2 m. Use the diagram below to complete the table.

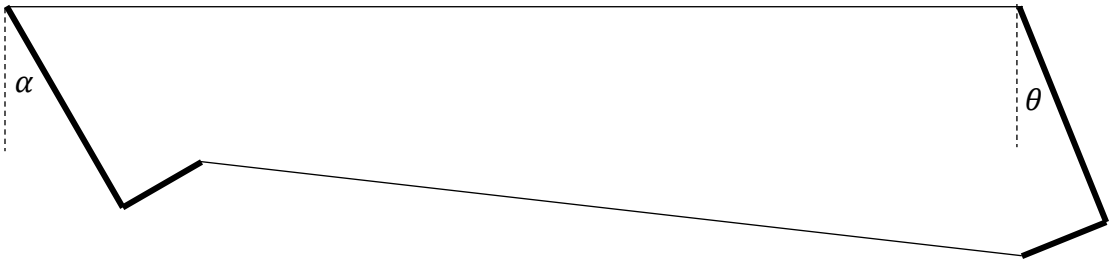
Angle of Wheel	
Outside Wheel (θ)	Inside Wheel (α)
0	
5	
10	
15	
20	
25	
30	



One steering system that will ensure the angle of the inside wheel is closer to the values in the table above is shown below. The front wheels are connected by a beam that is shorter in length than the distance between the wheels. This results in the wheels turning at different angles.



A simplified representation of this system is shown in the diagram below. When the outside wheel is turned by an angle of θ the inside wheel turns by an angle of α. Use the diagram to complete the table.



Angle of Wheel	
Outside Wheel (θ)	Inside Wheel (α)
0	
5	
10	
15	
20	
25	
30	

We need to change measurements in the steering design so that the differences between the values of α in this table and the perfect values of α in the first table are minimized.

Use this as a starting point for an investigation. Extend it in any way you wish…

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: o attempt to explain what is being investigated and how it is being investigated
3 – 4	The student is able to: I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure.	The student is able to o attempt to explain what is being investigated and how it is being investigated o use diagrams to justify some explanations and calculations o create a report that is able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure.	The student is able to o explain what is being investigated and how it is being investigated (including extending the given scenario) o use diagrams to justify explanations and calculations o move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure	The student is able to o clearly explain what is being investigated and how it is being investigated (including extending the given scenario) o display formulae clearly and accurately using the equation editor o use clear and accurate diagrams to justify explanations and calculations o move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) o make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) o create a report that is able to be understood without referring to the task sheet

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to o attempt to use appropriate mathematical tools and strategies to investigate the given scenario
3 – 4	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real-life situation IV. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to investigate the given scenario
5 – 6	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o use appropriate mathematical tools and strategies to accurately investigate the given scenario o give a good attempt at using appropriate mathematical tools and strategies to investigate an extension of the given scenario to a degree of rigour not below that of the given scenario
7 – 8	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation IV. justify the degree of accuracy of the solution V. justify whether the solution makes sense in the context of the authentic real-life situation.	The student is able to o make thorough use of appropriate mathematical tools and strategies to accurately investigate the given scenario o make thorough use of appropriate mathematical tools and strategies to accurately investigate an extension of the given scenario to a degree of rigour not below that of the given scenario