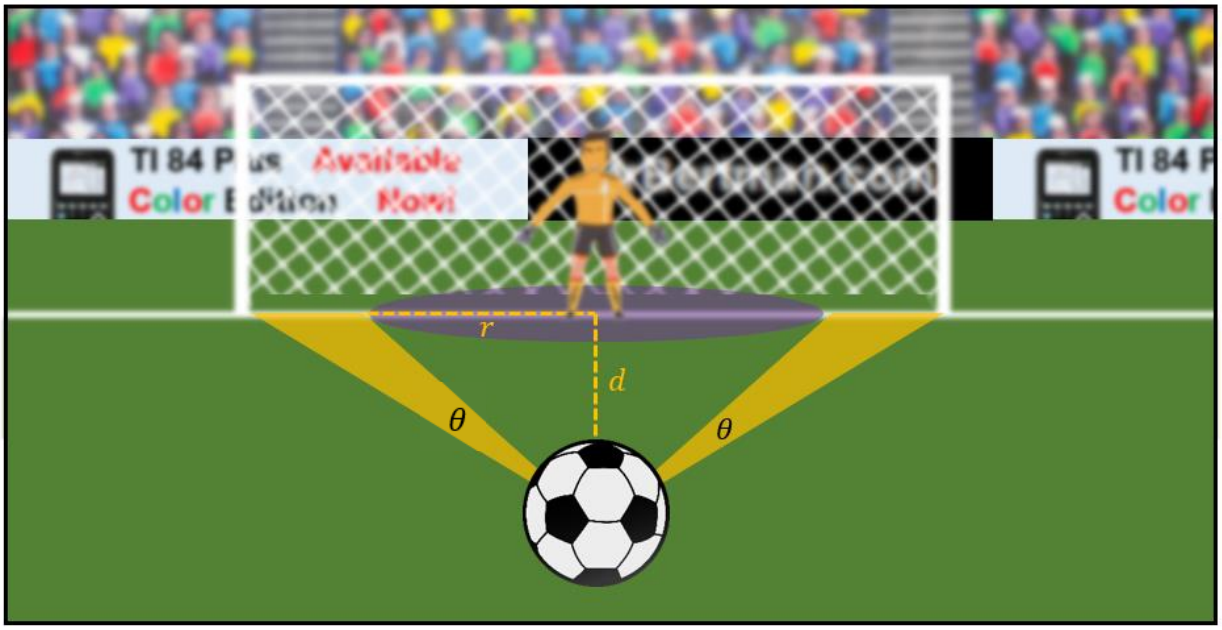
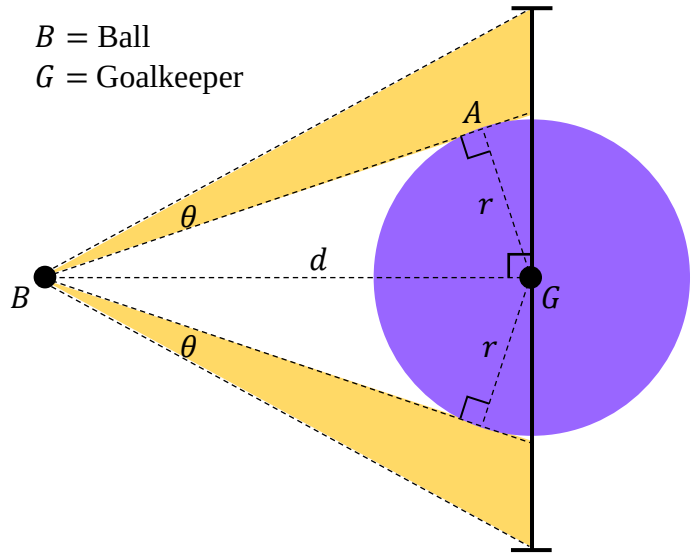
When Should I Shoot?

When playing football, if I shoot when I am far away from the goalkeeper he/she will have more time to react. If I shoot when I am too close to the goalkeeper it will be harder to avoid him/her. Assuming the goalkeeper stands in the centre of the goal, what is the best distance from which to shoot?



Consider the above diagram. The value of r, the radius of the area in which the goalkeeper can intercept the ball, depends on the value of d, the distance the ball is from the goalkeeper when a shot is taken. The larger the value of d then the larger the value of r. To have the best chance of scoring we need to maximize the value of θ. A two-dimensional view from above is shown below. The width of the goal is 7.32 m.

Let's assume that if the ball gets within 1 metre of the goalkeeper then the ball is definitely intercepted, we kick the ball at 20 m/s and the goalkeeper can move at 4 m/s.



Suppose it takes t seconds for the ball to reach point A

1. Write down an expression in terms of t for the length of AB.

2. Explain why $r = 4t + 1$.

3. Use the Pythagorean theorem to determine the relationship between t and d. Hence determine an expression for t in terms of d.

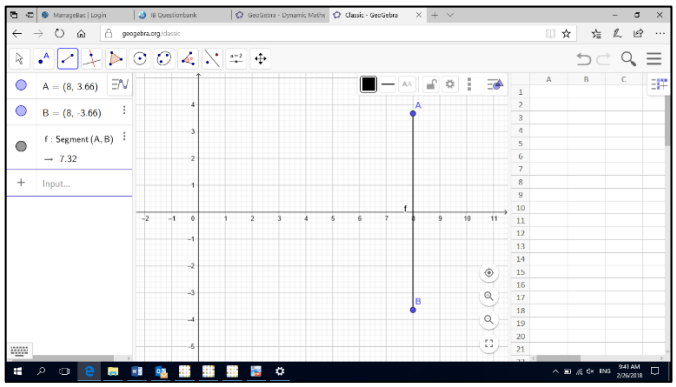
4. Determine an expression for angle θ in terms of only d.

5. Find the value of d that maximizes the value of θ.

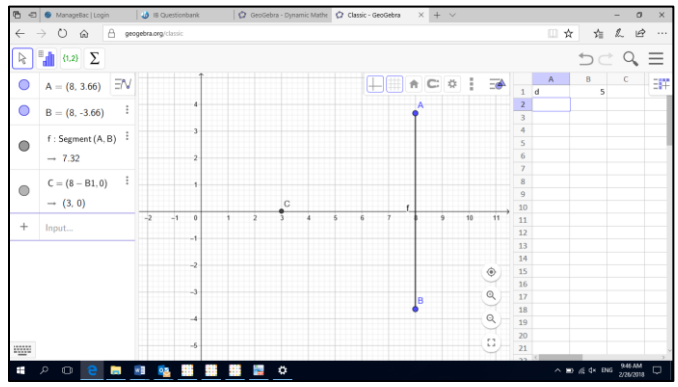
Use this as a starting point for an investigation. Extend it in any way you wish…

You may wish to use *Geogebra* to help you investigate.

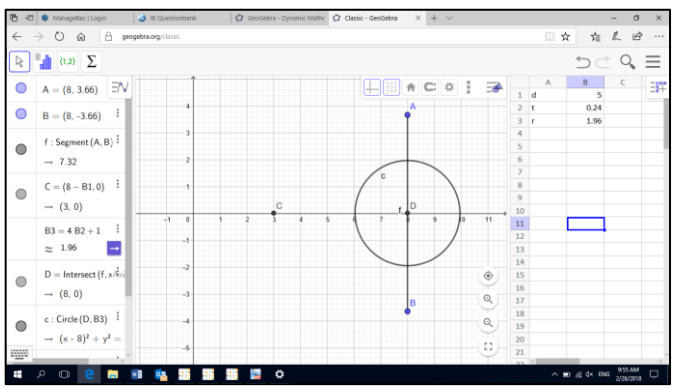
Step 1: Draw the goal



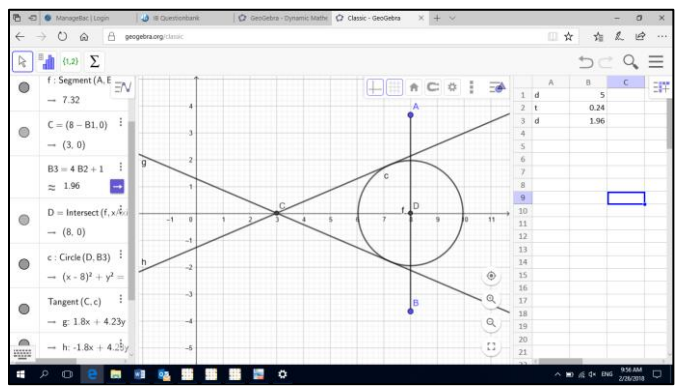
Step 2: Enter a value for distance *d* in the spreadsheet and position the player depending on this value (changing the value of *d* should automatically change the position of the player).



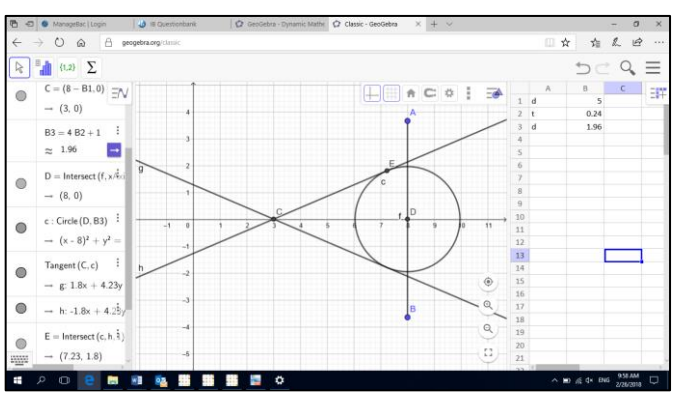
Step 3: In the spreadsheet calculate the values of *t* and *r* (use formulae, just as you would in an Excel spreadsheet). Create a circle of radius *r* showing where the goalkeeper may move to intercept the ball (changing the value of *d* in the spreadsheet should automatically update the values of *t* and *r*, and the size of the circle in the diagram).



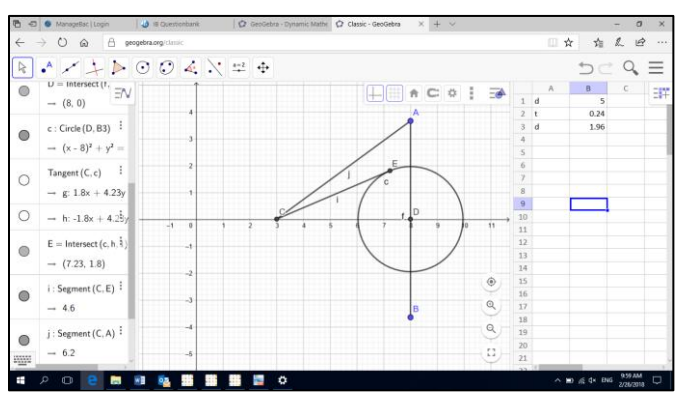
Step 4: Draw tangent lines from the player to the circle.



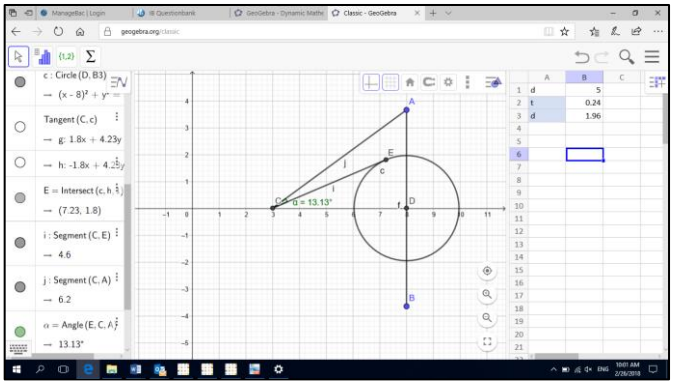
Step 5: Place a point where the tangent line meets the circle (using the *Point on Object* tool).



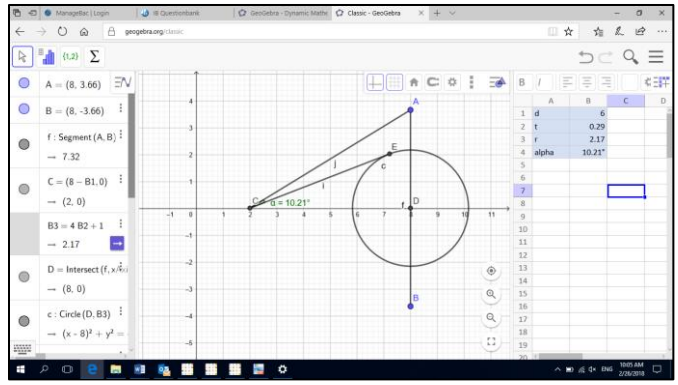
Step 6: Add some more line segments, hide lines you don't need.



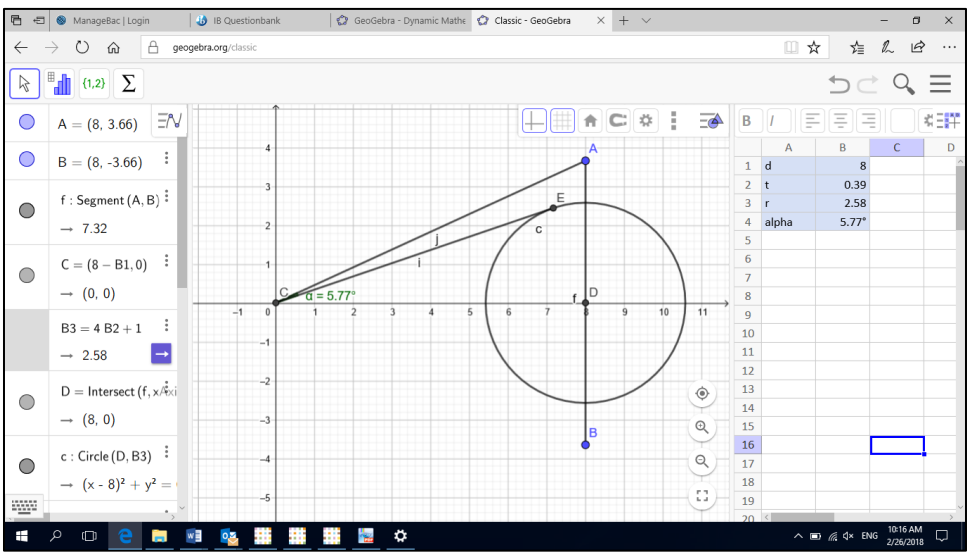
Step 7: Calculate angle *ECA*.

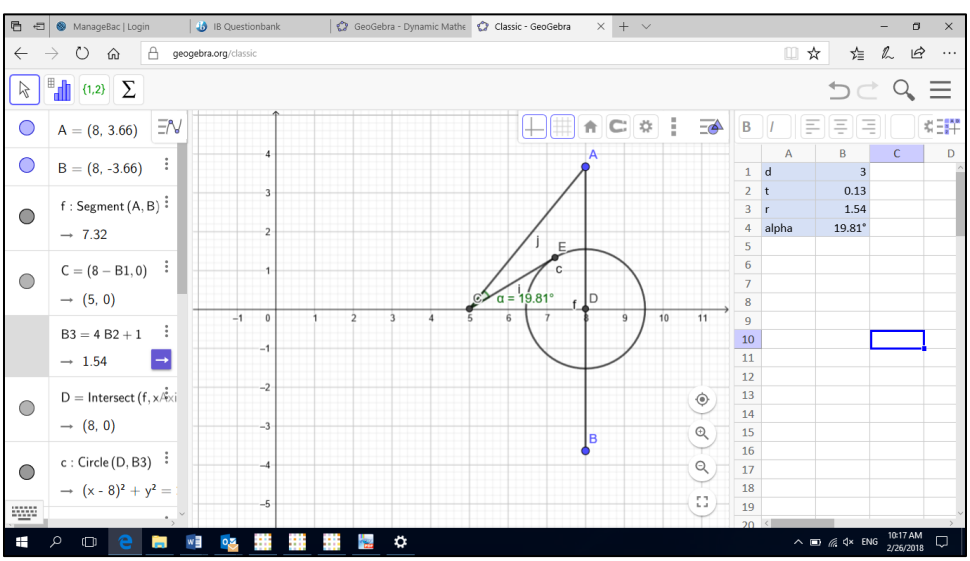


Step 8: Display the size of angle *ECA* in the spreadsheet. Now, change the value of *d* and everything should be updated automatically.



Check the accuracy of your sheet against the screenshots below:





Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: ○ attempt to explain what is being investigated and how it is being investigated
3 – 4	The student is able to: I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure.	The student is able to ○ attempt to explain what is being investigated and how it is being investigated ○ use diagrams to justify some explanations and calculations ○ create a report that is able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure.	The student is able to ○ explain what is being investigated and how it is being investigated ○ use diagrams to justify explanations and calculations ○ move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) ○ create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure	The student is able to ○ clearly explain what is being investigated and how it is being investigated ○ display formulae clearly and accurately using the equation editor ○ use clear and accurate diagrams to justify explanations and calculations ○ move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) ○ make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) ○ create a report that is able to be understood without referring to the task sheet

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to ○ attempt to use appropriate mathematical tools to determine approximate values of some results before attempting to calculate their exact values ○ attempt to determine the best position from which to shoot in the given model
3 – 4	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real-life situation IV. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to ○ use appropriate mathematical tools to try to determine approximate values of some results before attempting to calculate their exact values ○ determine the best position from which to shoot in the given model
5 – 6	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to ○ derive/justify any equations used ○ use appropriate mathematical tools to determine approximate values of some results before calculating their exact values ○ accurately determine the best position from which to shoot in the given model ○ investigate further situations which add to the complexity of the given model
7 – 8	The student is able to: I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation IV. justify the degree of accuracy of the solution V. justify whether the solution makes sense in the context of the authentic real-life situation.	The student is able to ○ derive/justify any equations used ○ justify the rejection or acceptance of any results / methods ○ use appropriate mathematical tools to thoroughly investigate scenarios and determine approximate values of all results before calculating their exact values ○ clearly and accurately determine the best position from which to shoot in the given model ○ clearly, accurately and thoroughly investigate further situations which add to the complexity of the given model