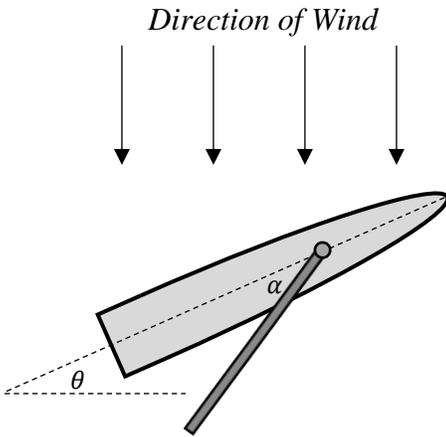


# Sailing Against the Wind



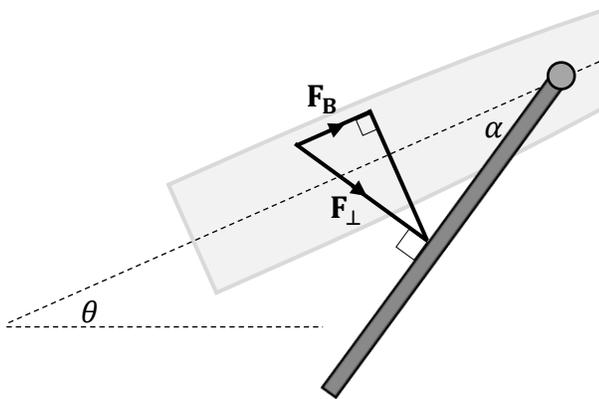
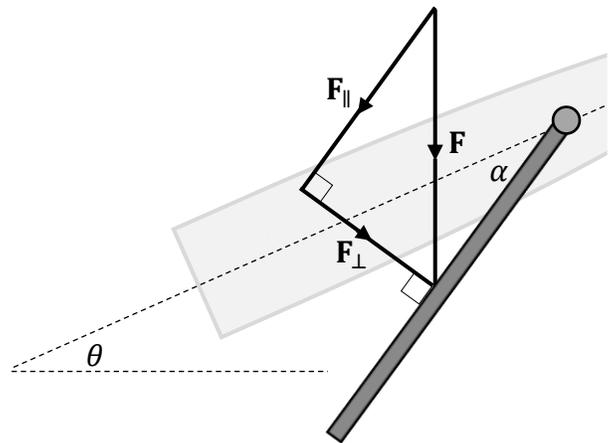
Although it is impossible for a sailboat to sail directly into the wind, it is possible for it to zig-zag to a destination which is upwind.

Consider the diagram on the left representing a sailboat from above. The wind is blowing from north to south. The angle between the direction in which the boat is pointing and due east is  $\theta$ . The angle between the sail and the direction in which the boat is pointing is  $\alpha$ .

The wind exerts a force on the sail. The sail then exerts a force on the boat. Suppose the boat needs to reach a destination which is directly north. What values of  $\theta$  and  $\alpha$  will enable the boat to reach its destination in the shortest amount of time?

Let the force of the wind on the sails be equal to  $\mathbf{F}$ . Some of this force will act parallel to the sail and some will act perpendicular to the sail. Let these be represented by  $\mathbf{F}_{\parallel}$  and  $\mathbf{F}_{\perp}$  respectively. This is shown in the diagram on the right.

- Determine an expression for the angle between  $\mathbf{F}$  and  $\mathbf{F}_{\perp}$  in terms of  $\theta$  and  $\alpha$ .
- Determine an expression for  $\mathbf{F}_{\perp}$  in terms of  $\mathbf{F}$ ,  $\theta$  and  $\alpha$ .

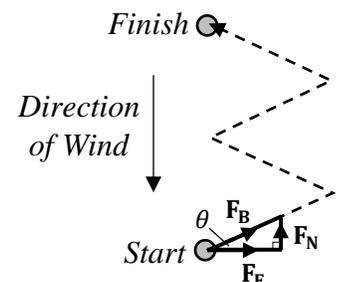


The sail will exert a force on the boat with a force equal to  $\mathbf{F}_{\perp}$ . Some of this will act parallel and some will act perpendicular to the direction in which the boat is facing. It is the parallel force that will move the boat forwards. Let this force be represented by  $\mathbf{F}_{\mathbf{B}}$ . This is shown in the diagram on the left.

- Determine an expression for the angle between  $\mathbf{F}_{\mathbf{B}}$  and  $\mathbf{F}_{\perp}$  in terms of  $\theta$  and  $\alpha$ .
- Determine an expression for  $\mathbf{F}_{\mathbf{B}}$  in terms of  $\mathbf{F}$ ,  $\theta$  and  $\alpha$ .

The sailboat is pushed forwards through the water by force  $\mathbf{F}_{\mathbf{B}}$ . Some of this force is acting to the north  $\mathbf{F}_{\mathbf{N}}$ , and some is acting to the east  $\mathbf{F}_{\mathbf{E}}$ . It is the force that is acting to the north that we need to maximize in order to zig-zag to our destination in the shortest amount of time.

Use this as a starting point for an investigation. Extend it in any way you wish...



Criterion C: Communication in Mathematics

Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	<p>The student is able to:</p> <ul style="list-style-type: none"> <li>I. use limited mathematical language</li> <li>II. use limited forms of mathematical representation to present information</li> <li>III. communicate through lines of reasoning that are difficult to interpret.</li> </ul>	<p>The student is able to:</p> <ul style="list-style-type: none"> <li>○ attempt to explain what is being investigated and how it is being investigated</li> </ul>
3 – 4	<p>The student is able to:</p> <ul style="list-style-type: none"> <li>I. use some appropriate mathematical language</li> <li>II. use appropriate forms of mathematical representation to present information adequately</li> <li>III. communicate through lines of reasoning that are complete</li> <li>IV. adequately organize information using a logical structure.</li> </ul>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ attempt to explain what is being investigated and how it is being investigated</li> <li>○ use diagrams to justify some explanations and calculations</li> <li>○ create a report that is able to be understood without referring to the task sheet</li> </ul>
5 – 6	<p>The student is able to:</p> <ul style="list-style-type: none"> <li>I. usually use appropriate mathematical language</li> <li>II. usually use appropriate forms of mathematical representation to present information correctly</li> <li>III. usually move between different forms of mathematical representation</li> <li>IV. communicate through lines of reasoning that are complete and coherent</li> <li>V. present work that is usually organized using a logical structure.</li> </ul>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ explain what is being investigated and how it is being investigated (including extending the given scenario)</li> <li>○ use diagrams to justify explanations and calculations</li> <li>○ move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.)</li> <li>○ create a report that is able to be understood without referring to the task sheet</li> </ul>
7 – 8	<p>The student is able to:</p> <ul style="list-style-type: none"> <li>I. consistently use appropriate mathematical language</li> <li>II. use appropriate forms of mathematical representation to consistently present information correctly</li> <li>III. move effectively between different forms of mathematical representation</li> <li>IV. communicate through lines of reasoning that are complete, coherent and concise</li> <li>V. present work that is consistently organized using a logical structure</li> </ul>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ clearly explain what is being investigated and how it is being investigated (including extending the given scenario)</li> <li>○ display formulae clearly and accurately using the equation editor</li> <li>○ use clear and accurate diagrams to justify explanations and calculations</li> <li>○ move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.)</li> <li>○ make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.)</li> <li>○ create a report that is able to be understood without referring to the task sheet</li> </ul>

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	<p>The student is able to:</p> <ol style="list-style-type: none"> <li>I. identify some of the elements of the authentic real-life situation</li> <li>II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.</li> </ol>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ attempt to use appropriate mathematical tools and strategies to investigate the given scenario</li> </ul>
3 – 4	<p>The student is able to:</p> <ol style="list-style-type: none"> <li>I. identify the relevant elements of the authentic real-life situation</li> <li>II. select, with some success, adequate mathematical strategies to model the authentic real-life situation</li> <li>III. apply mathematical strategies to reach a solution to the authentic real-life situation</li> <li>IV. discuss whether the solution makes sense in the context of the authentic real-life situation.</li> </ol>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ use appropriate mathematical tools and strategies to investigate the given scenario</li> </ul>
5 – 6	<p>The student is able to:</p> <ol style="list-style-type: none"> <li>I. identify the relevant elements of the authentic real-life situation</li> <li>II. select adequate mathematical strategies to model the authentic real-life situation</li> <li>III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation</li> <li>IV. explain the degree of accuracy of the solution</li> <li>V. explain whether the solution makes sense in the context of the authentic real-life situation.</li> </ol>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ use appropriate mathematical tools and strategies to accurately investigate the given scenario</li> <li>○ give a good attempt at using appropriate mathematical tools and strategies to investigate an extension of the given scenario to a degree of rigour not below that of the given scenario</li> </ul>
7 – 8	<p>The student is able to:</p> <ol style="list-style-type: none"> <li>I. identify the relevant elements of the authentic real-life situation</li> <li>II. select appropriate mathematical strategies to model the authentic real-life situation</li> <li>III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation</li> <li>IV. justify the degree of accuracy of the solution</li> <li>V. justify whether the solution makes sense in the context of the authentic real-life situation.</li> </ol>	<p>The student is able to</p> <ul style="list-style-type: none"> <li>○ make thorough use of appropriate mathematical tools and strategies to accurately investigate the given scenario</li> <li>○ make thorough use of appropriate mathematical tools and strategies to accurately investigate an extension of the given scenario to a degree of rigour not below that of the given scenario</li> </ul>