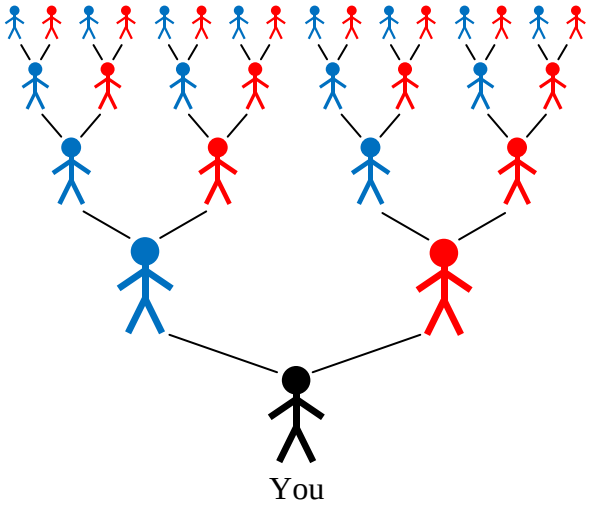
Are We All Related?

The population of the earth in 2016 is approximately 7.4 billion. It is the greatest that it has ever been in the history of the world.

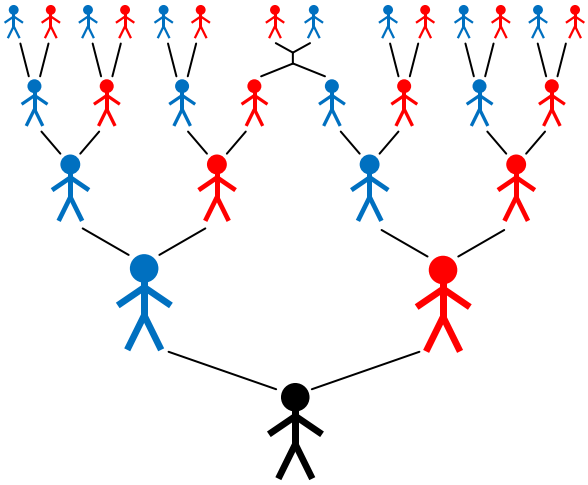


Consider your own family tree. You have two parents. Your parents each have two parents (your grandparents). Your grandparents each have two parents (your great-grandparents) etc.

If you continue this pattern back for 50 generations what is the problem with this model? *Answer this question before you continue.*

The problem is caused by assuming all members of a generation in your family tree are unrelated.

The following diagram shows one possible way in which all members of a generation in your family tree are not unrelated.



In this model your parents are related to each other. They are (distant) cousins since they share at least one ancestor. Also, two of your great-grandparents are siblings.

Notice that you only have 14 great-great-grandparents instead of 16 as in the previous model. This reduces the size of all earlier generations.

However, if we assume that all members of all earlier generations are unrelated we will still run into the same problem as before…

Task 1

Suppose the initial population of a species is 100000 newborns, the life expectancy is 5 years and the birth rate is 10%.

Year	Age of members of population					Total
	1	2	3	4	5	
1	100000	0	0	0	0	100000
2	10000	100000	0	0	0	110000
3	11000	10000	100000	0	0	121000
4	12100	11000	10000	100000	0	133100
5	13310	12100	11000	10000	100000	146410
6	14641	13310	12100	11000	10000	61051

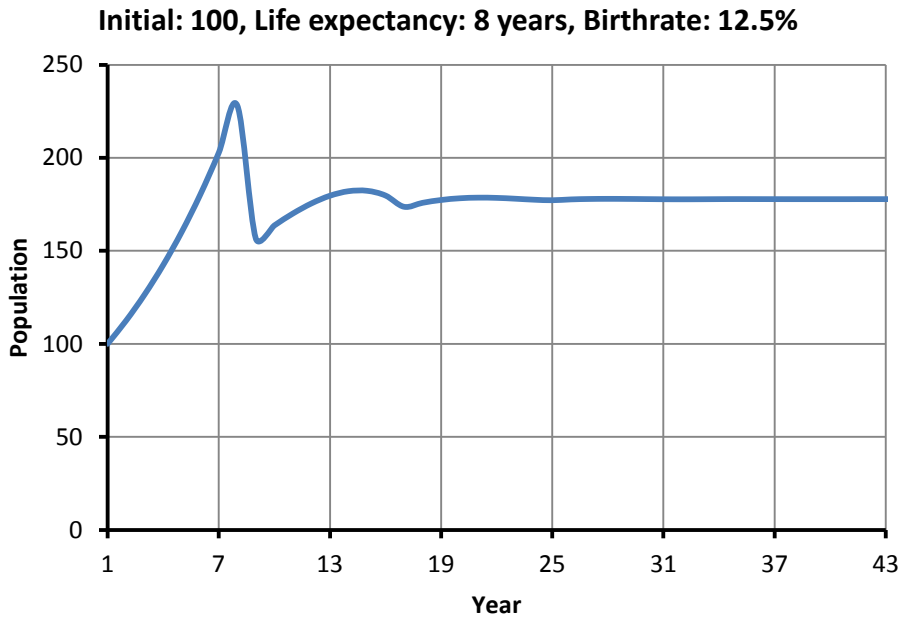
If this model is continued will the population increase, become stable, or become extinct?

Investigate the relationship between initial population, birth rate, life expectancy and population growth. Under which conditions will a population increase, become stable, and become extinct?

Task 2

In the case of a stable population in task 1, perform an investigation to determine how we can calculate the value of the stable population given only the initial population and life expectancy?

For example when the initial population is 100, the life expectancy is 8 years and the birth-rate is 12.5% the population eventually becomes stable with a value of $177.\dot{7}$.



Task 3

Assume that the population of a planet has been stable for a *very* long time. You may decide the value for the population (choose any value between 1 million and 1 billion). Assume you live on this planet.

- The length of one generation is approximately 25 years. Determine the approximate maximum number of years ago when one of your ancestors must've reproduced with a relative.
- Determine the minimum number of family members in the row of your family tree 1000 generations ago in the following cases:
  - none of your relatives reproduced with a sibling and every relative only reproduced with one partner.
  - none of your relatives reproduced with a sibling or a first cousin and every relative only reproduced with one partner.
  - none of your relatives reproduced with a sibling, a first cousin, or a second cousin and every relative only reproduced with one partner.
  - none of your relatives reproduced with a sibling, a first cousin, a second cousin, a third cousin, …, or an n^{th} cousin and every relative only reproduced with one partner.
- *Thoroughly* discuss the question "are we all related"?

Submit your work as a report in PDF format. Be sure to include an introduction. The word limit is 1000. Anything exceeding this will not be graded. All information has been provided so you do not and should not perform any outside research. Everything you submit must have been created by you, using technology.

Definitions

Grandparent	*a parent of a parent*
Great-grandparent	*a parent of a grandparent*
Great-great-grandparent	*a parent of a great-grandparent*
Siblings	*two or more people who share at least one parent*
Cousins	*two or more people who are not siblings but share at least one ancestor*
First cousins	*two or more people who are not siblings but share at least one grandparent*
Second cousins	*two or more people who are not siblings or first cousins but share at least one great-grandparent*
Third cousins	*two or more people who are not siblings, first cousins or second cousins but share at least one great-great-grandparent*

Criterion B: Investigating Patterns		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **apply, with teacher support**, mathematical problem-solving techniques to discover **simple patterns** II. **state predictions** consistent with patterns.	The student is able to: o investigate population growth for various initial populations, life expectancies and birth rates in task 1
3 – 4	The student is able to: I. **apply** mathematical problem-solving techniques to discover **simple patterns** II. **suggest general rules** consistent with **findings**.	The student is able to: o determine under which conditions a population will increase, become stable, and become extinct in task 1
5 – 6	The student is able to: I. **select and apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **findings** III. **verify** the validity of these general rules.	The student is able to: o determine the value of any stable population using only the initial population and life expectancy in task 2 o determine the maximum number of years ago when one of your ancestors must've reproduced with a relative in task 3
7 – 8	The student is able to: I. **select** and **apply** mathematical problem-solving techniques to discover **complex patterns** II. **describe patterns** as general rules consistent with **correct findings** III. **prove, or verify and justify**, these general rules.	The student is able to: o determine the minimum number of family members in the row of your family tree 1000 generations ago for the various cases in task 3

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **use** limited mathematical language II. **use** limited forms of mathematical representation to present information III. **communicate** through lines of reasoning that are difficult to interpret.	The student is able to: ○ attempt to develop population models in task 1 ○ attempt to explain the problem with assuming every member of a generation is unrelated and the solution to this problem
3 – 4	The student is able to: I. **use** some appropriate mathematical language II. **use** different forms of mathematical representation to present information adequately III. **communicate** through lines of reasoning that are able to be understood, although these are not always clear IV. adequately **organize** information using a logical structure.	The student is able to: ○ develop population models in task 1 ○ attempt to create family trees in task 3 ○ attempt to discuss the question "are we all related?" ○ explain the problem with assuming every member of a generation is unrelated and the solution to this problem
5 – 6	The student is able to: I. usually **use** appropriate mathematical language II. usually **use** different forms of mathematical representation to present information correctly III. move between different forms of mathematical representation with some success IV. **communicate** through lines of reasoning that are clear although not always coherent or complete V. present work that is usually **organized** using a logical structure.	The student is able to: ○ use tables to develop an accurate population model in task 1 ○ use graphs to demonstrate growing populations, stable populations and declining populations in task 1 and stable populations in task 2 ○ create family trees in task 3 ○ thoroughly discuss the question "are we all related?" ○ explain the problem with assuming every member of a generation is unrelated and the solution to this problem ○ create a report that is able to be understood without referring to the task sheet
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use different forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is consistently organized using a logical structure.	The student is able to: ○ use tables to develop an accurate population model in task 1 ○ use appropriately formatted and labelled graphs to demonstrate growing populations, stable populations and declining populations in task 1 and stable populations in task 2 ○ use the equation editor for all equations ○ create clear and accurate family trees in task 3 ○ thoroughly discuss the question "are we all related?" using results from the investigation and any further relevant discoveries ○ clearly explain the problem with assuming every member of a generation is unrelated, using appropriate diagrams and mathematics, and the solution to this problem ○ create a report that is able to be understood without referring to the task sheet