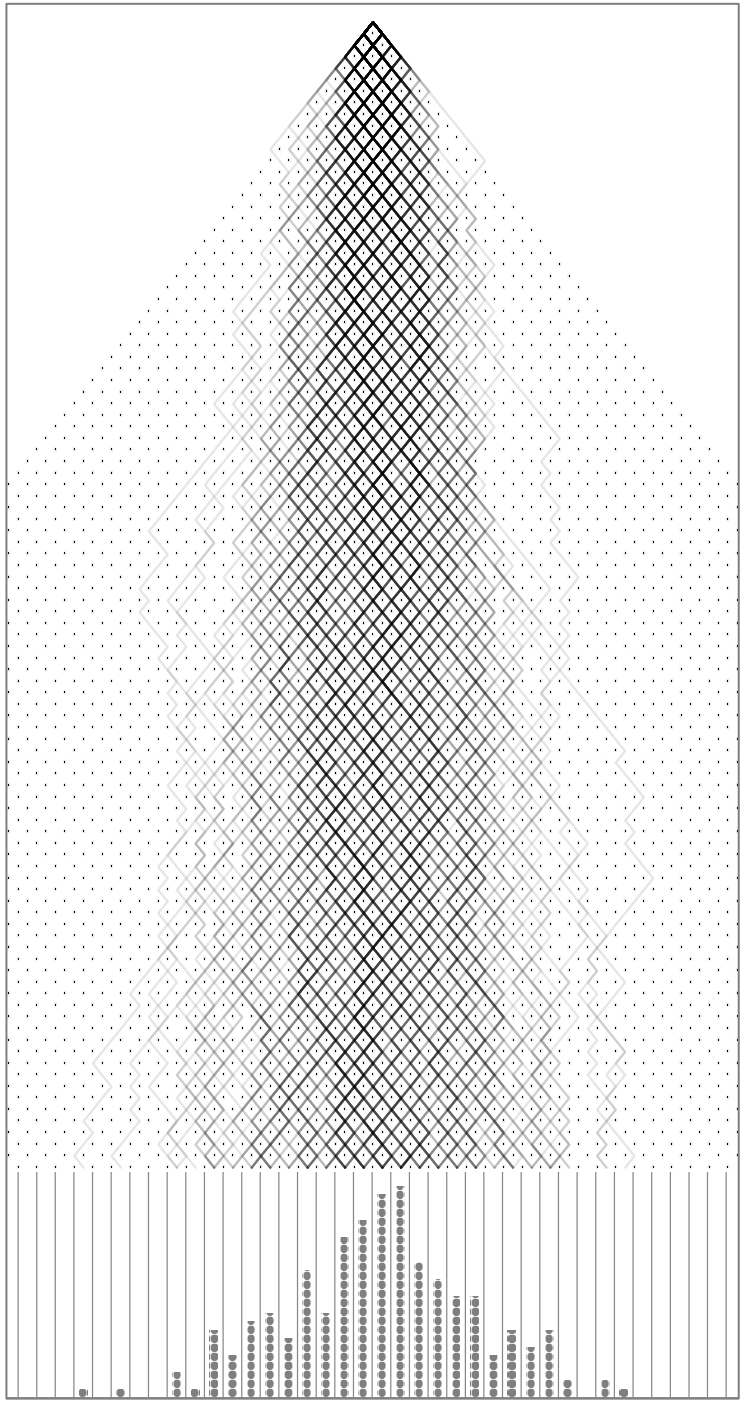
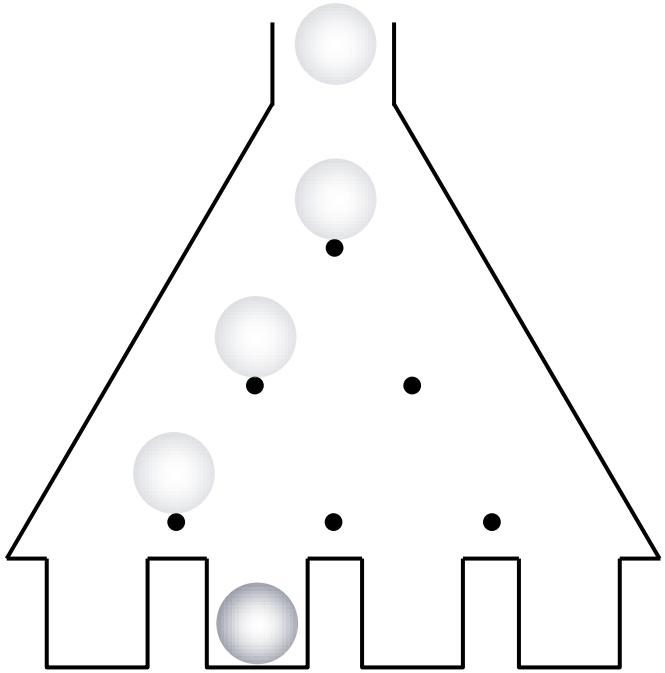


Approximating the Normal Distribution

Continuous data which is created by the combined effects of many independent random variables is often normally distributed.

We can approximate the normal distribution using discrete data if the sample space is large.

Consider the following machine. A ball is dropped into the top and at each pin it has an equal chance of going left or right. Eventually the ball falls into a cup.



The diagram on the left shows a much larger version of this machine. The lines show the paths of 245 balls dropped into the machine.

Task 1

Consider a machine with five cups. Using the flip of a coin (or similar) to determine the direction of a ball at each pin, determine how many balls will be in each cup if forty balls are used.

Hint: First draw the machine to determine how many pins it must have. You will have to flip the coin once for each row of pins you have.

Task 2

Consider a very small machine with two cups. Determine the probability of a ball landing in each cup. Repeat this for machines with three, four and five cups. Use any patterns you notice to derive an expression for the probability of landing in the i^{th} cup from the left in a machine with n cups.

Hint: Write the probabilities as fractions but do not simplify them. This will help you to identify any patterns.

Task 3

Use Excel to perform an experiment dropping 10,000 balls into a machine with 1,000 cups. Plot a column graph showing the results of the experiment.

Hint: Use =RANDBETWEEN(0,1) to determine the direction of a ball at each pin where 0 represents left and 1 represents right. This will help you with the next task.

Other useful functions (for tasks 3 and 4) may be: IF, SUM, COUNTIF, AVERAGE, STDEV.P, COMBIN

Task 4

If the cups in task 3 are numbered from 0 to 999 determine the mean and standard deviation of the numbers on the cups in which the 10,000 balls landed. By performing relevant calculations determine how well the results of the experiment fit the normal distribution.

Hint: Your random numbers will keep changing every time you edit your Excel file. If you wish to stop this from happening highlight them all, copy them, then click *Paste* > *Values* on the *Home* tab.

When working with continuous data we have $P(X < x) = P(X \leq x)$ (i.e. it makes no difference if we use $<$ or $\leq$). This is not the case with discrete data. You will have to decide how to deal with this.

Task 5

If 10,000 balls are dropped into a machine with 1,000 cups determine the expected number of balls in each cup. If the cups are numbered from 0 to 999 perform relevant calculations to determine how well this data fits the normal distribution.

Hint: This is a purely theoretical task (i.e. you shouldn't use your Excel model). However, the numbers involved get very large so you may wish to use Excel for any calculations you have.

Create a report of your work no longer than 1000 words. The report should be able to stand on its own without the reader having to reference the task sheet. All work should be created electronically by you.

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **use** limited mathematical language II. **use** limited forms of mathematical representation to present information III. **communicate** through lines of reasoning that are difficult to interpret.	The student is able to: ○ attempt to explain what is being investigated and how it is being investigated ○ create a report that is partly able to be understood without referring to the task sheet
3 – 4	The student is able to: I. **use** some appropriate mathematical language II. **use** different forms of mathematical representation to present information adequately III. **communicate** through lines of reasoning that are able to be understood, although these are not always clear IV. adequately **organize** information using a logical structure.	The student is able to: ○ explain what is being investigated and how it is being investigated without the reader having to refer to the task sheet ○ use diagrams/screenshots to complement explanations and calculations ○ create a report that is partly able to be understood without referring to the task sheet
5 – 6	The student is able to: I. usually **use** appropriate mathematical language II. usually **use** different forms of mathematical representation to present information correctly III. move between different forms of mathematical representation with some success IV. **communicate** through lines of reasoning that are clear although not always coherent or complete V. present work that is usually **organized** using a logical structure.	The student is able to: ○ explain what is being investigated and how it is being investigated (including explanations of how any spreadsheets work) ○ display formulae using the equation editor ○ use diagrams/screenshots to complement explanations and calculations ○ create a report that is mostly able to be understood without referring to the task sheet ○ use mostly appropriate mathematical notation
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use different forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is consistently organized using a logical structure.	The student is able to: ○ clearly explain in his/her own words what is being investigated and how it is being investigated (including explanations of how any spreadsheets work) ○ display formulae clearly and accurately using the equation editor ○ use clear and accurate diagrams/screenshots to complement explanations and calculations ○ move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 3 demonstrates, etc.) ○ make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) ○ create a report that is able to be understood without referring to the task sheet ○ use appropriate mathematical notation

Criterion D: Applying Mathematics in Real Life Contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. **identify** some of the elements of the authentic real-life situation II. **apply** mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to: o use a coin (or similar) to simulate dropping 40 balls through a machine with 5 cups
3 – 4	The student is able to: I. **identify** the relevant elements of the authentic real-life situation II. **select**, with some success, adequate mathematical strategies to model the authentic real-life situation III. **apply** mathematical strategies to reach a solution to the authentic real life situation IV. **describe** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: o determine the theoretical probability of landing in each cup in a machine with two, three, four and five cups
5 – 6	The student is able to: I. **identify** the relevant elements of the authentic real-life situation II. **select** adequate mathematical strategies to model the authentic real-life situation III. **apply** the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. **describe** the degree of accuracy of the solution V. **discuss** whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: o derive the theoretical probability of landing in the i^{th} cup from the left in a machine with n cups o develop an excel model to simulate dropping 10,000 balls through a machine with 1,000 cups
7 – 8	The student is able to: I. **identify** the relevant elements of the authentic real-life situation II. **select** appropriate mathematical strategies to model the authentic real life situation III. **apply** the selected mathematical strategies to reach a correct solution IV. **explain** the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the **authentic** real-life situation.	The student is able to: o determine the expected number of balls in each cup when 10,000 balls are dropped through a machine with 1,000 cups. o thoroughly discuss, using relevant calculations, how well both the experimental model and the theoretical model fit the normal distribution