Climbing Mountains
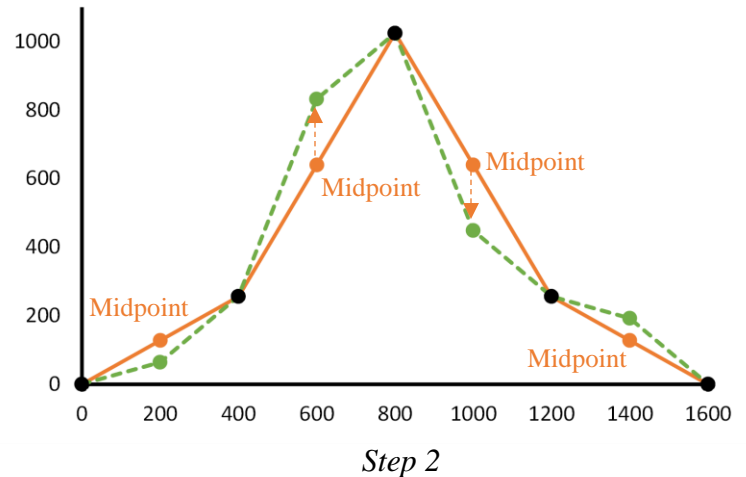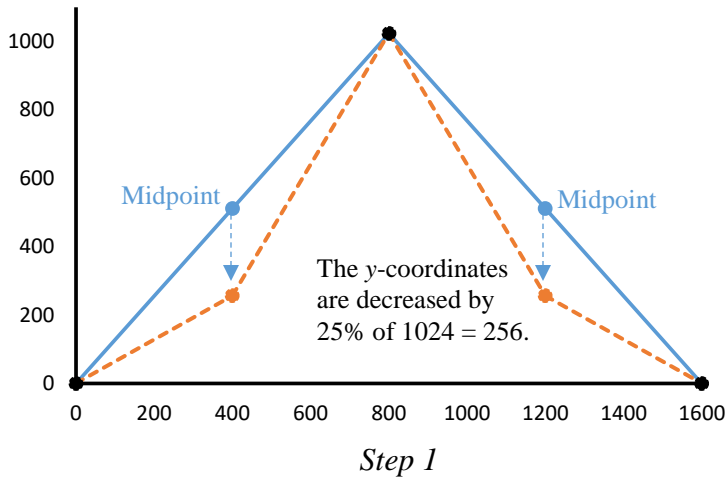
Two fixed points with coordinates (0,0) and (1600,0) are connected to the fixed point (800,1024) with straight lines. The *y*-coordinate of the midpoint of each of these lines is then randomly increased or decreased by 25% of the height of each segment. These new points become fixed. This is shown in step 1 below.

Step 1

Step 2

The process is repeated for each of the four red line segments. The *y*-coordinate of the midpoint of each segment is randomly increased or decreased by 25% of the height of the segment. For example:

- the *y*-coordinate of the midpoint of the first segment is decreased by 64
- the *y*-coordinate of the midpoint of the second segment is increased by 192
- the *y*-coordinate of the midpoint of the third segment is decreased by 192
- the *y*-coordinate of the midpoint of the fourth segment is increased by 64

Check you understand how these values are calculated before you move on.

These new points become fixed. This is shown in step 2 above. All black points in the diagrams are fixed.

This process can be repeated for as many steps as you like. This is one way to generate realistic looking terrain using a mathematical algorithm. The large image below shows the above mountain after 4 steps, along with other mountains in the background. The smaller images were also generated using the same algorithm.
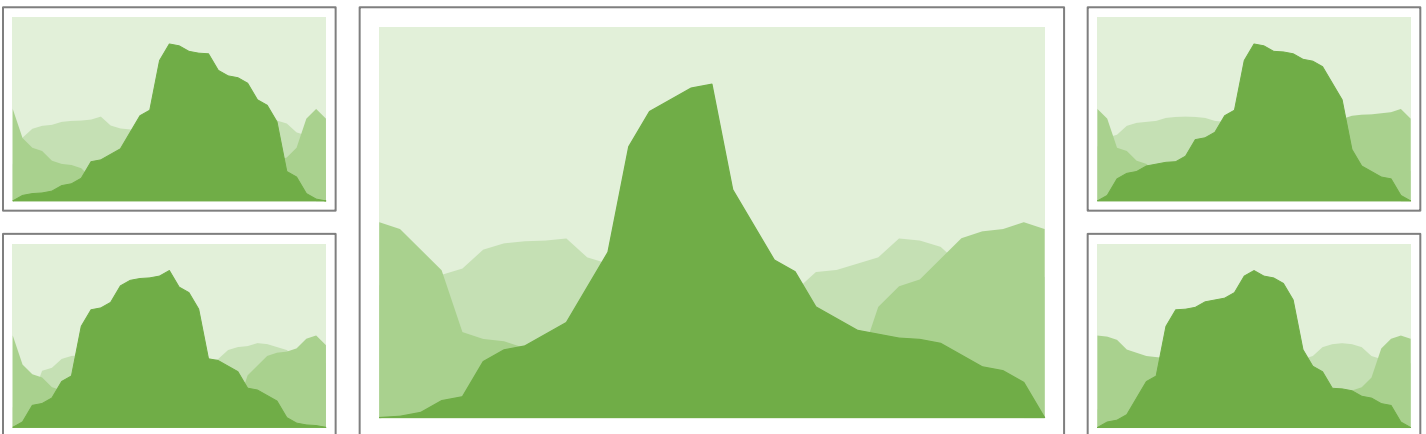
How far would a person have to hike/climb to reach the top of a mountain created by performing the algorithm up to step 5? How about up to step *n*? Create your own mountain and investigate. Create a report of your findings.

Excel Tutorial

Set up your spreadsheet as in the screenshot on the right. This values in columns A and B represent the x and y coordinates of the first two line segments.

It is important that you enter the values in exactly the same cells as in the screenshot.

To practice creating graphs you may wish to highlight cells A3 to B5 and select:

Insert > Charts > Scatter with Straight Lines

	A	B	C	D	E	F
1	Step 0			Step 1		
2	x	y	Random	x	y	
3	0	0				
4	800	1024				
5	1600	0				
6						
7						
8						
9						

Remember that the coordinates of the midpoint between two points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

D	E
Step 1	
x	y
=A3	=B3

The point (0,0) is fixed, so we need it in step 1. Enter the formulae shown in the screenshot on the left into cells D3 and E3.

When you press *enter* the values of cells A3 and B3 will appear.

In cell D4 we need the x-coordinate of the midpoint of (0,0) and (800,1024). We can apply the midpoint formula by typing the formula shown in the screenshot on the right into cell D4.

When you press *enter* this coordinate will be calculated.

D	E
Step 1	
x	y
=A3	=B3
=(A3+A4)/2	

In cell E4 we need to calculate the y-coordinate of the midpoint of (0,0) and (800,1024) and randomly increase it or decrease it by 25% of 1024, the height of the line segment.

D	E
Step 1	
x	y
=A3	=B3
=(A3+A4)/2	=(B3+B4)/2

Select cell D4 (select cells with a single click), copy it (Ctrl + c), select cell E4, and paste (Ctrl + v). Notice what happens to the pasted formula. We pasted it one cell to the right, so the column reference changes from an A to a B (i.e. it increases by one).

This value in cell E4 is now the y-coordinate of the midpoint of (0,0) and (800,1024).

We need to randomly increase or decrease the y-coordinate by 25% of 1024, the height of the line segment. To do this we first need to generate a random number. This number will determine whether we will increase or decrease the y-coordinate. The function RANDBETWEEN(a,b) will calculate a random integer between the values of a and b inclusive.

In cell C4 enter the equation as shown in the screenshot on the right. This will determine a random integer of either 0 or 1, multiply it by 2 (so it becomes 0 or 2), then subtract 1 (so it becomes -1 or 1).

C	
Random	x
	=A3
=2*RANDBETWEEN(0,1)-1	=(A3

We can use this random number to randomy increase or decrease the *y*-coordinate of the midpoint. If the number is 1 the coordinate will increase and if it is -1 it will decrease. Modify the contents of cell E4 as in the screenshot on the right.

This formula multiplies 25% of the height of the line segment by 1 or -1 and adds this value to the *y*-coodinate of the midpoint.

E
y
=B3
=(B3+B4)/2+C4*(B4-B3)*0.25

	D	E
		Step 1
	x	y
	=A3	=B3
	=(A3+A4)/2	=(B3+B4)/2+C4*(B4-B3)*0.
	=A4	=B4

The next point is the fixed point located at (800,1024). Enter the formulae shown in the screenshot on the left in cells D5 and E5.

You may wish to try copying and pasting cells D3 and E3 to cells D5 and E5 instead. What happens?

Cell D6 represents the *x*-coordinate of the midpoint of the second segment. Cell E6 represents the *y*-coordinate of the midpoint of the second segment plus or minus 25% of the height of the segment.

Highlight cells C4, D4 and E4, copy (Ctrl + c), highlight cells C6, D6 and E6 an paste (Ctrl + v).

Notice what happens to the formulae. We pasted the formulae two cells below from where they were copied. This means all row reference numbers increase by 2. This is not correct.

C	D	E
		Step 1
Random	x	y
	=A3	=B3
=2*RANDBETWEEN(0,1)-1	=(A3+A4)/2	=(B3+B4)/2+C4*(B4-B3)*0.25
	=A4	=B4
=2*RANDBETWEEN(0,1)-1	=(A5+A6)/2	=(B5+B6)/2+C6*(B6-B5)*0.25

	D	E
		Step 1
	x	y
	=A3	=B3
1	=(A3+A4)/2	=(B3+B4)/2+C4*(B4-B3)*0.25
	=A4	=B4
1	=(A4+A5)/2	=(B4+B5)/2+C6*(B5-B4)*0.25

For example cell D6 should use cells A4 and A5 to calculate the midpoint, not cells A5 and A6. Cell E6 should use cells B4 and B5, not cells B5 and B6.

Modify the formulae as shown in the screenshot on the left.

The final point in step 1 is the fixed point located at (1600,0). Enter the formulae shown in the screenshot on the right.

Step 1 is now complete. See what it looks like by highlighting cells D3 to E7 and selecting:

Insert > Charts > Scatter with Straight Lines

	D	E
		Step 1
	x	y
	=A3	=B3
	=(A3+A4)/2	=(B3+B4)/2+C4*(B4-B3)*0.25
	=A4	=B4
	=(A4+A5)/2	=(B4+B5)/2+C6*(B4-B5)*0.25
	=A5	=B5

You now have enough knowledge to continue creating further steps…

Criterion B: Investigating Patterns		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below.	
1 – 2	The student is able to: I. apply, with teacher support, mathematical problem-solving techniques to discover simple patterns II. state predictions consistent with patterns.	The student is able to: o determine how far a person has to hike/climb to reach the top of the mountain after step 1 and 2
3 – 4	The student is able to: I. apply mathematical problem-solving techniques to discover simple patterns II. suggest general rules consistent with findings.	The student is able to: o determine how far a person has to hike/climb to reach the top of the mountain after step 1, 2 and 3 o suggest any patterns in these three steps that may help to calculate further steps
5 – 6	The student is able to: I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with findings III. verify the validity of these general rules.	The student is able to: o determine how far a person has to hike/climb to reach the top of the mountain after step 1, 2, 3 and 4 o develop the suggested pattern into a rule that can be used to calculate further steps
7 – 8	The student is able to: I. select and apply mathematical problem-solving techniques to discover complex patterns II. describe patterns as general rules consistent with correct findings III. prove, or verify and justify, these general rules.	The student is able to: o use the developed rule to determine how far a person has to climb to reach the top of the mountain after step 5 o determine how far a person has to hike/climb to reach the top of the mountain after step n

Criterion C: Communication in Mathematics		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	The student is able to: I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret.	The student is able to: ○ attempt to explain what is being investigated and how it is being investigated in the student's own words ○ create mountains of various steps using the algorithm
3 – 4	The student is able to: I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure.	The student is able to ○ adequately explain what is being investigated and how it is being investigated in the student's own words ○ create and use diagrams to justify/clarify some explanations and calculations ○ create a report that is able to be understood without referring to the task sheet ○ create mountains of various steps using the algorithm up to step 5
5 – 6	The student is able to: I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure.	The student is able to ○ clearly explain what is being investigated and how it is being investigated in the student's own words ○ display formulae clearly and accurately using the equation editor ○ create and use diagrams to justify/clarify explanations and calculations ○ move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following diagram shows, figure 2 demonstrates etc.) ○ create a report that is able to be understood without referring to the task sheet ○ create mountains of various steps using the algorithm up to step 5
7 – 8	The student is able to: I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure	The student is able to ○ clearly explain what is being investigated and how it is being investigated in the student's own words ○ display formulae clearly and accurately using the equation editor ○ create and use clear and accurate diagrams to justify/clarify explanations and calculations ○ move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) ○ make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) ○ create a report that is able to be understood without referring to the task sheet ○ create mountains of various steps using the algorithm up to step 5 ○ celebrate the beauty of mathematics by creating a mountain scene containing several mountains using the algorithm up to step 5