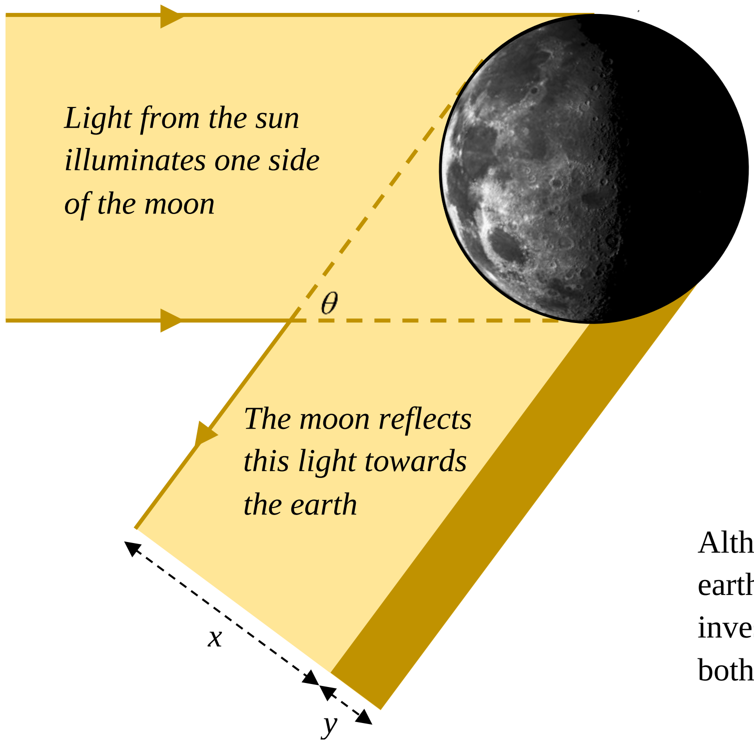
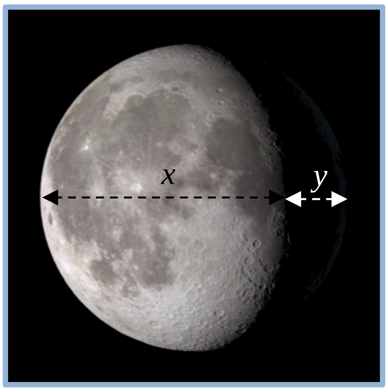
# Determining the Phases of the Moon

The earth orbits the sun in approximately 365.25 days. The moon orbits the earth in approximately 27.3 days. The diagram below shows why we see different phases of the moon depending on the size of angle $\theta$ which changes according to the relative positions of the sun, earth and moon. (*Images of the moon taken from http://www.nasa.org*).



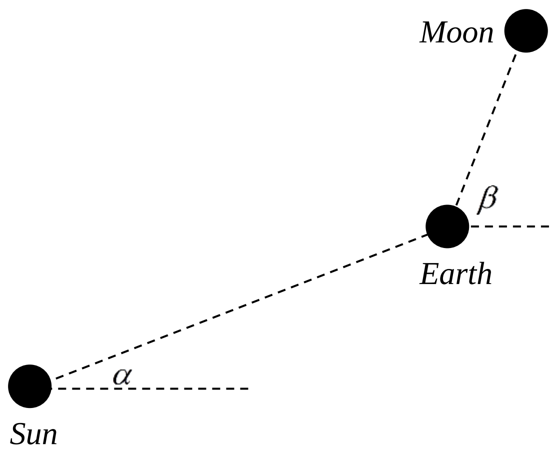

*The view as seen from earth*

Although the orbits of both the moon and the earth are slightly elliptical, throughout this investigation you may assume that they are both circular.

*Task 1*

Determine expressions for the values of $x$ and $y$ in terms of $\theta$ and the radius of the moon $r$. Hence, determine an expression for the ratio $x : y$ in the simplest terms.

The following diagram represents the view of the sun, earth and moon from above after $n$ days. Initially (when $n = 0$) both $\alpha$ and $\beta$ are equal to zero, resulting in a full moon. All orbits are anti-clockwise. The diagram is not to scale.




You may assume that the distance between the sun and earth is $1.495 \times 10^8$ km and the distance between earth and the moon is $3.844 \times 10^5$ km.

*Task 2*

By modifying the diagram on the left, label angle $\theta$ along with the two distances from above.

*Task 3*

Determine the ratio $x : y$ for $n = 0,1,2,3,...$ Use a spreadsheet to help you organize your calculations.

*Hint: Since the ratio* $x : y$ depends on the value of $\theta$ *you will have to find this angle. This can be done using the cosine rule, but the cosine rule uses the lengths of all three sides of the triangle…*

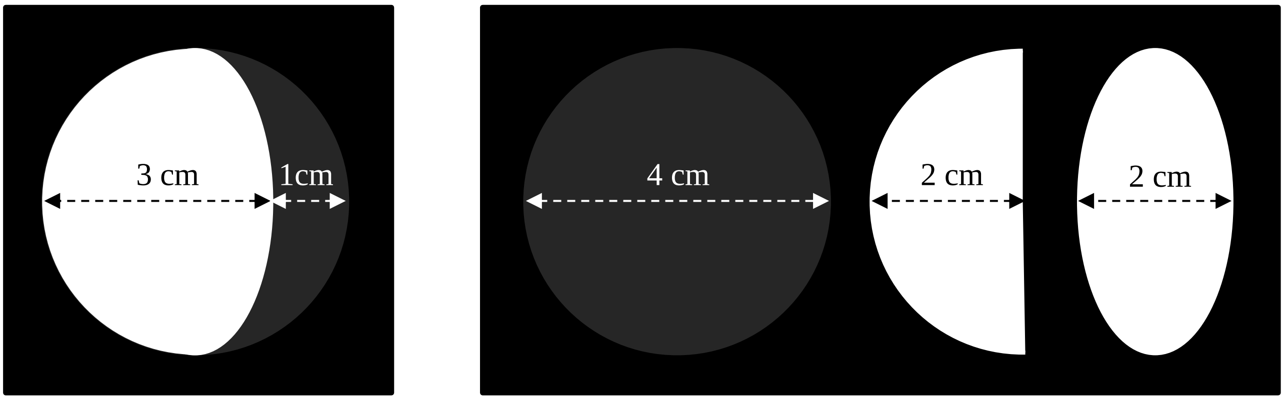
*Task 4*

- Test the accuracy of your model for a date which is far into the future. For example, compare your model to published lunar calendars. Is your model accurate? Explain.
- Use your model to determine the amount of time between identical phases of the moon. Why is this time not equal to 27.3 days?

Create a report of your word. Be sure to include:

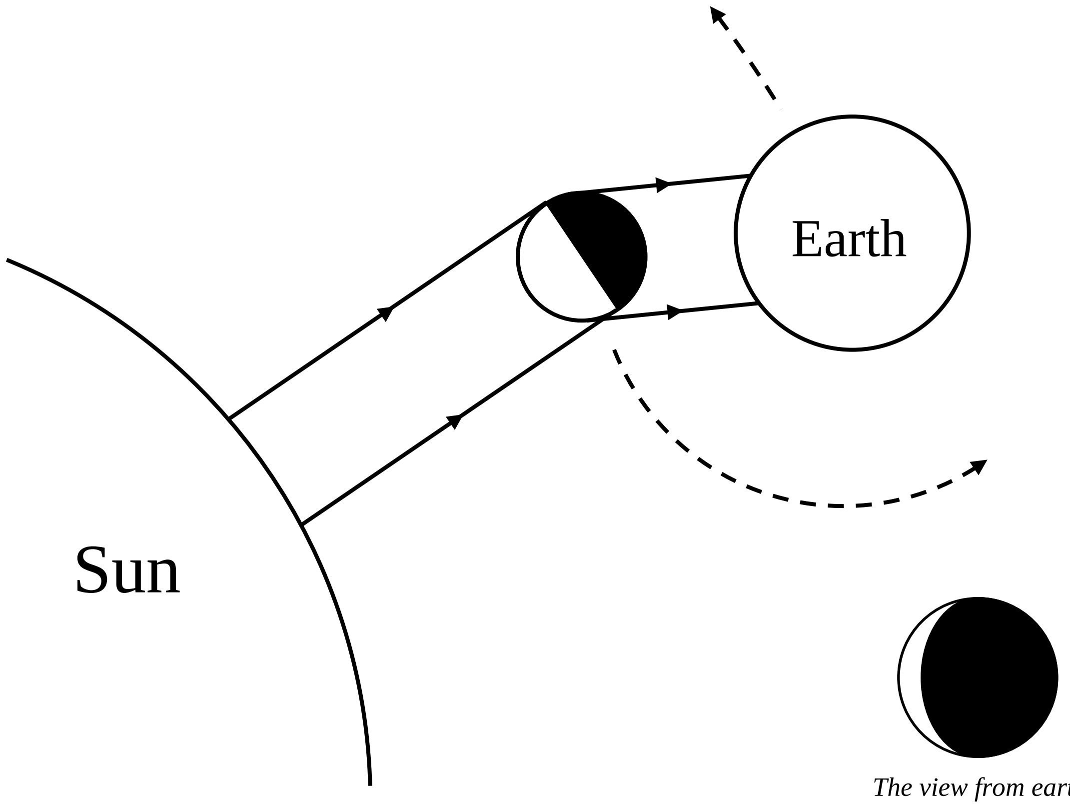
- an explanation of why we see different phases of the moon
- the derivation of how you calculated the phase for any value of *n*
- your solutions to task 4
- some moon phases based on data from your model

*Tip: You can create diagrams showing phases of the moon in any simple graphics application, or using the drawing tools in Word. For example, the diagram on the left is created using three shapes:*



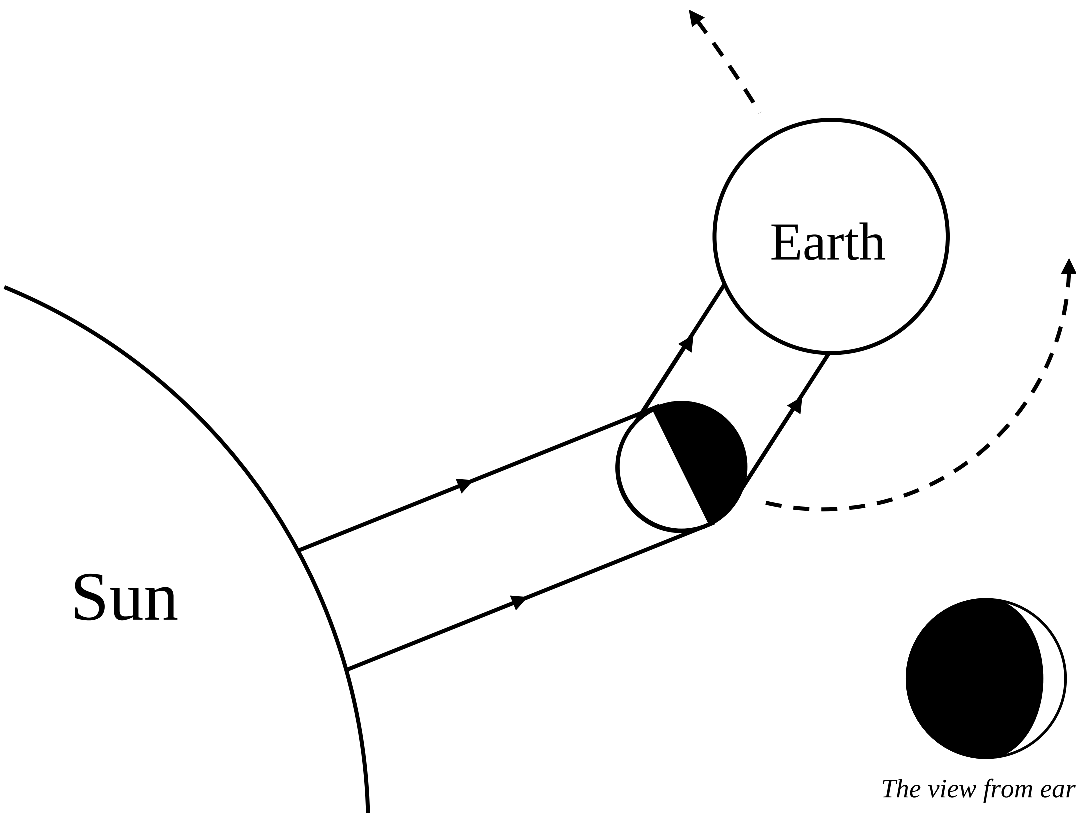
*IMPORTANT POINT*

Once the moon has passed beyond a certain point, the position of *x* and *y* swap. In the first diagram *x* appears on the left side of the moon.




*The view from earth*

After a little more time, *x* appears on the right side of the moon.

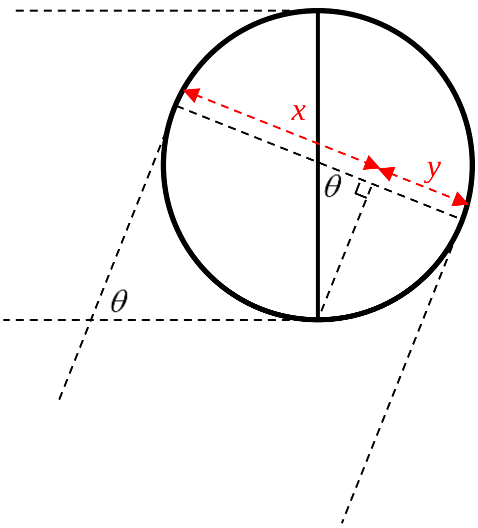



*The view from earth*

This first happens on day 15 i.e. when the value of $\beta - \alpha$ first exceeds $\pi$ radians. The positions of *x* and *y* will then swap back to their original positions when the value of $\beta - \alpha$ first exceeds $2\pi$ radians. This is repeated as the earth and moon continue their orbits.

# Teacher Notes

*Task 1*




If the radius of the moon is $r$ then

$$x = r + r\cos\theta$$

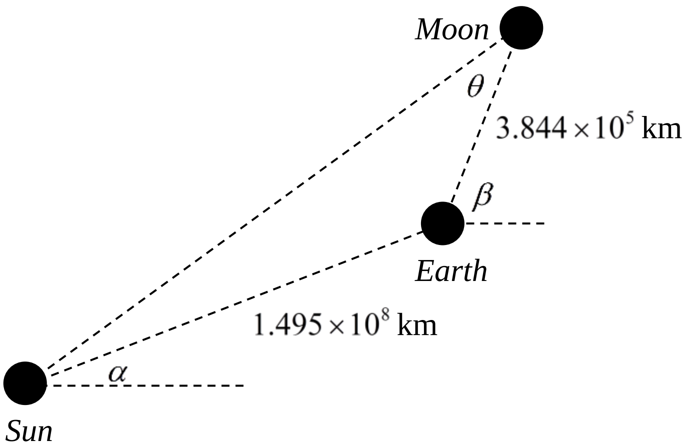
and

$$y = r - r\cos\theta$$

So we have

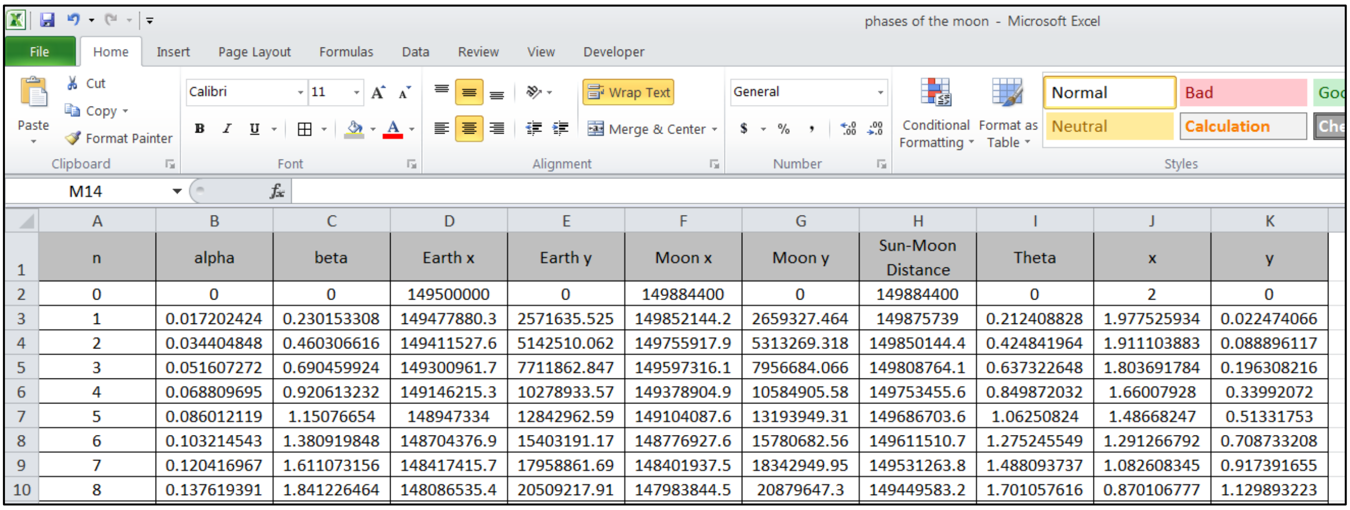
$$x : y = 1 + \cos\theta : 1 - \cos\theta$$

*Task 2*




*Task 3*

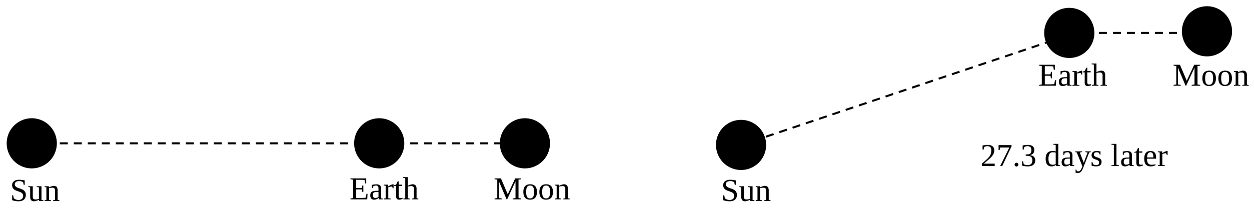
Let the distance from the sun to the moon be equal to *d*. We can use trigonometry to determine the coordinates of earth and the moon. We can then use the Pythagorean Theorem to determine the distance from the sun to the moon. We can then use the cosine rule to determine angle $\theta$, allowing us to calculate the ratio $x : y$. The first few rows of the spreadsheet are below. Note that when using trigonometric functions in Excel we must use radians. Hence, all angles are in radians.



| n | alpha | beta | Earth x | Earth y | Moon x | Moon y | Sun-Moon Distance | Theta | x | y |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 149500000 | 0 | 149884400 | 0 | 149884400 | 0 | 2 | 0 |
| 1 | 0.017202424 | 0.230153308 | 149477880.3 | 2571635.525 | 149852144.2 | 2659327.464 | 149875739 | 0.212408828 | 1.977525934 | 0.022474066 |
| 2 | 0.034404848 | 0.460306616 | 149411527.6 | 5142510.062 | 149755917.9 | 5313269.318 | 149850144.4 | 0.424841964 | 1.911103883 | 0.088896117 |
| 3 | 0.051607272 | 0.690459924 | 149300961.7 | 7711862.847 | 149597316.1 | 7956684.066 | 149808764.1 | 0.637322648 | 1.803691784 | 0.196308216 |
| 4 | 0.068809695 | 0.920613232 | 149146215.3 | 10278933.57 | 149378904.9 | 10584905.58 | 149753455.6 | 0.849872032 | 1.66007928 | 0.33992072 |
| 5 | 0.086012119 | 1.15076654 | 148947334 | 12842962.59 | 149104087.6 | 13193949.31 | 149686703.6 | 1.06250824 | 1.48668247 | 0.51331753 |
| 6 | 0.103214543 | 1.380919848 | 148704376.9 | 15403191.17 | 148776927.6 | 15780682.56 | 149611510.7 | 1.275245549 | 1.291266792 | 0.708733208 |
| 7 | 0.120416967 | 1.611073156 | 148417415.7 | 17958861.69 | 148401937.5 | 18342949.95 | 149531263.8 | 1.488093737 | 1.082608345 | 0.917391655 |
| 8 | 0.137619391 | 1.841226464 | 148086535.4 | 20509217.91 | 147983844.5 | 20879647.3 | 149449583.2 | 1.701057616 | 0.870106777 | 1.129893223 |

*Task 4*

Since the earth is rotating around the sun after 27.3 days the sun, earth and moon will not be in the same positions relative to each other as they were 27.3 days previously. This is demonstrated in the following diagrams (not to scale).




It will take a few more days for the sun, earth and moon to be in the same relative positions. The time taken for one orbit, 27.3 days, is called a *sidereal* month. The time taken between identical phases, approximately 29.5 days, is called a *lunar* month.

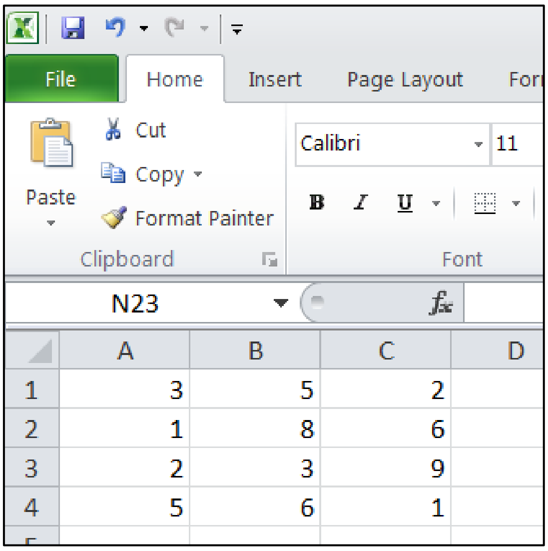
*Geogebra*

Geogebra has spreadsheet functionality which can be linked to the graphics view. This means we can easily create a diagram of any phase quite easily. See the provided Geogebra file for an example.

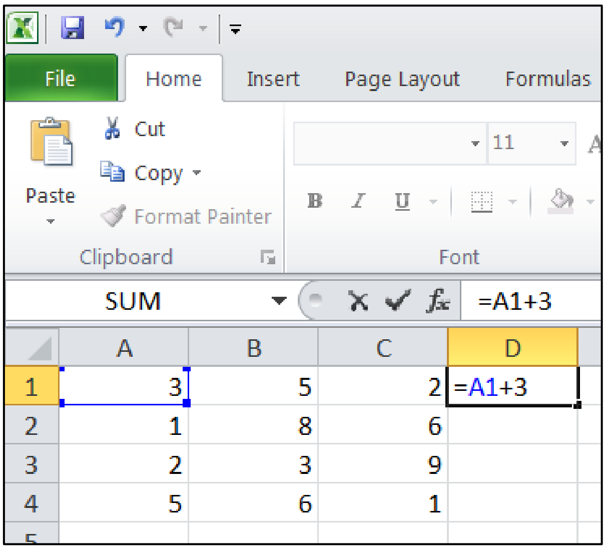
# Excel Tutorial

*Relative References*

Type the following values into cells A1 to C4:

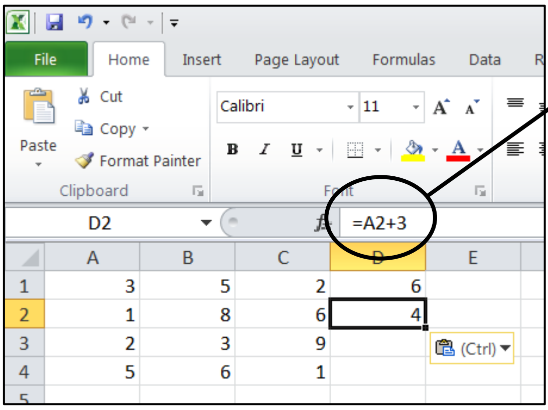



Type the following equation into cell D1:




Press *Enter* and the value displayed in cell D1 should be equal to 6.

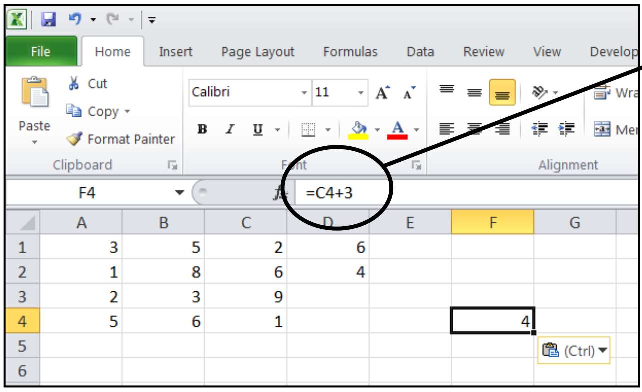
Copy and paste cell D1 to cell D2:




Notice that the equation in cell D2 becomes A2+3.

This is because we pasted one row below cell D1, so the reference A1 becomes A2 (the row number increases by one).

Copy and paste cell D1 into cell F4:



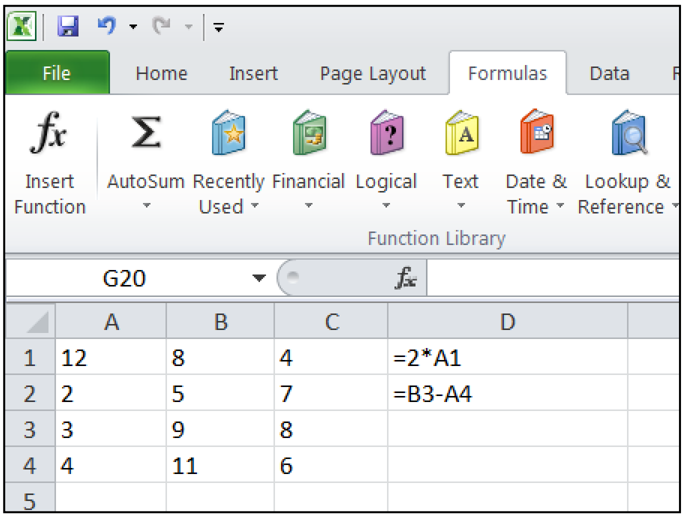

Notice that the equation in cell F4 becomes C4+3.

This is because we pasted two columns to the right and three rows below cell D1, so the reference A1 becomes C4 (the column letter increases by two and the row number increases by three).

These are examples of *relative references*.

*Task 1*

The following screenshot shows the contents of various cells (Note that you will not see the equations on your screen after you press *Enter*).




| | A | B | C | D |
|---|---|---|---|---|
| 1 | 12 | 8 | 4 | =2*A1 |
| 2 | 2 | 5 | 7 | =B3-A4 |
| 3 | 3 | 9 | 8 | |
| 4 | 4 | 11 | 6 | |
| 5 | | | | |

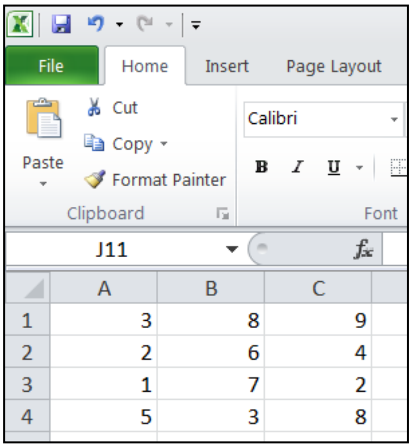
Complete the following table *without* using Excel. Then, check your answers using Excel.

For example, if we copy and paste cell D1 to cell E2, cell E2 will contain the equation 2*B2 and will display the value of 10.

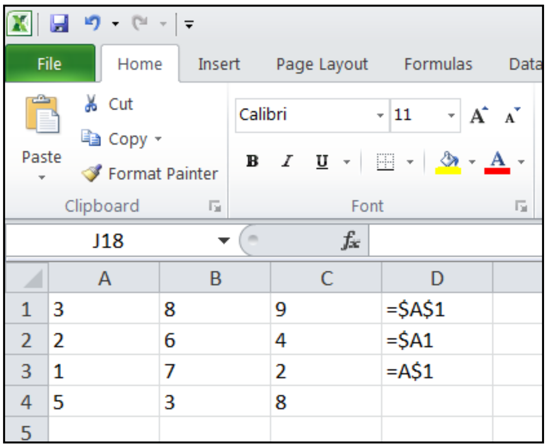
| Copied From | Pasted To | Displayed Equation | Displayed Value |
|---|---|---|---|
| D1 | E2 | 2*B2 | 10 |
| D1 | D4 | | |
| D1 | E3 | | |
| D2 | E1 | | |
| D2 | F2 | | |
| D2 | E3 | | |

*Absolute References*

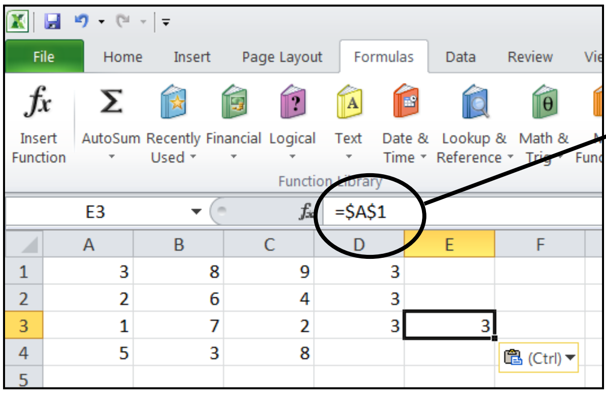
Type the following values in cells A1 to C4:




Type the following equations into cells D1 to D3:




Copy and paste cell D1 to E3:

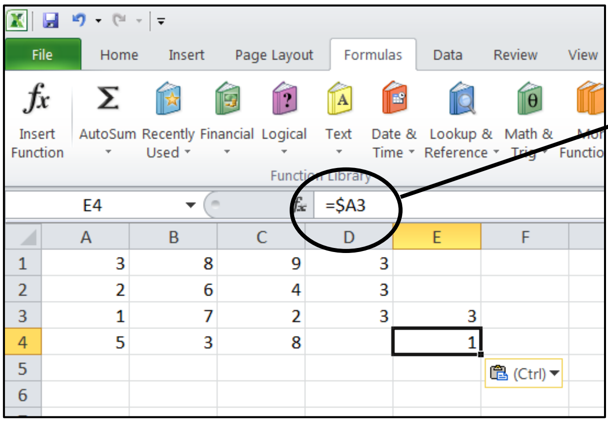



Notice that even though we pasted to a new column and row, the cell reference doesn't change.

Placing a $ sign in front of the column letter means the column letter will never change.

Placing a $ sign in front of the row number means the row number will never change.

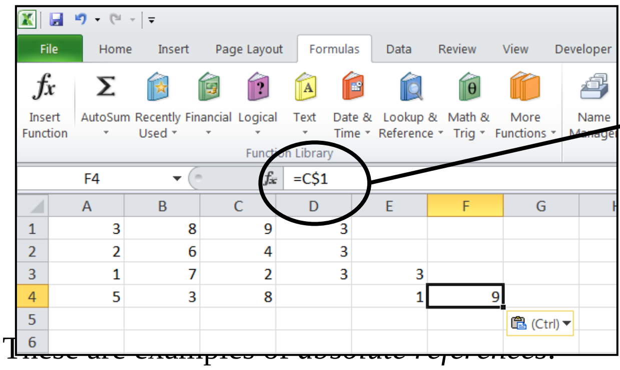
Copy and paste cell D2 to E4:




Since there is a $ sign in front of the column letter, the column letter will never change.

Cell E4 is two rows below cell D2, so the row number increases by two.

Copy and paste cell D3 into cell F4:



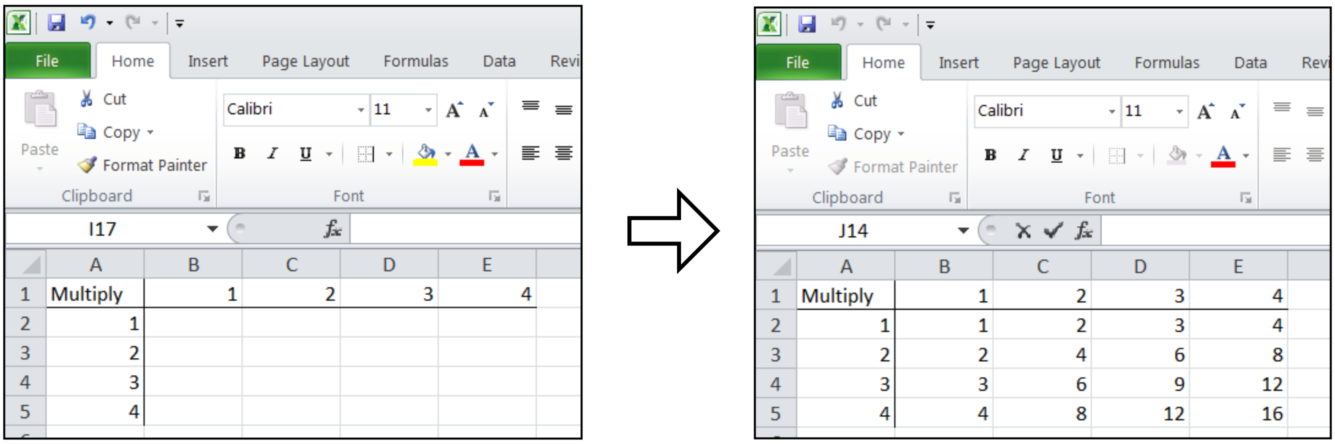

Since there is a $ sign in front of the row number, the row number will never change.

Cell F4 is two columns to the right of cell D3, so the column letter increases by two.

These are examples of absolute *references*.

*Task 2*

Enter one equation into cell B2, so that when it is copied and pasted into all of the other cells it correctly fills in the multiplication table:

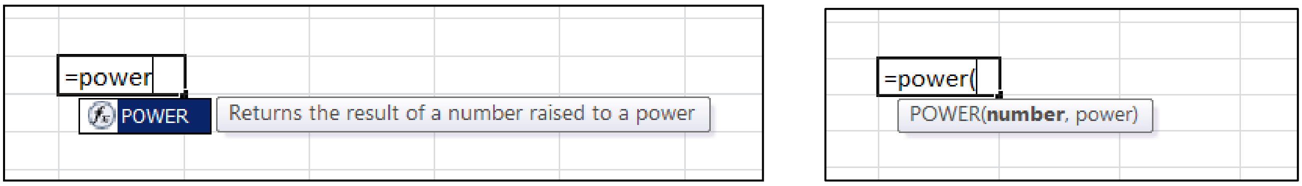



*Mathematical Functions*

Excel contains many mathematical functions. For example:

sin(), cos(), tan(), asin(), acos(), atan(), sqrt(), power(), int(), randbetween(), sum(), average(),countif()

As you type the function into Excel, you will often be given a description of the function and what kind of values it accepts:




Examples

- Typing =power(3,4) will display the value of 81
- Type =sqrt(9) will display the value of 3
- Typing =sum(A1:A6) will calculate the sum of the contents of cells A1 to A6

*Task 3*

Use Excel to determine the following:

a) the square root of 20
b) the mean value of 20, 18, 23, 11, 25 and 19
c) a randomly generated number between 5 and 13
d) the inverse sine of 1
e) the sum of all positive integers less than 10