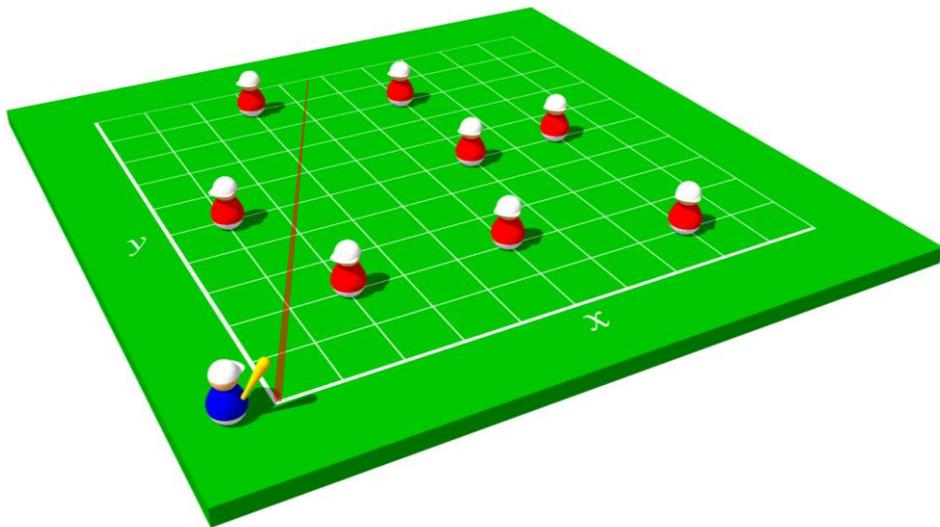


Where Should I Hit the Ball?

In some sports, for example baseball or volleyball, it is in the best interest of the person hitting the ball to hit it so that it has the least chance of being intercepted by the opposition.



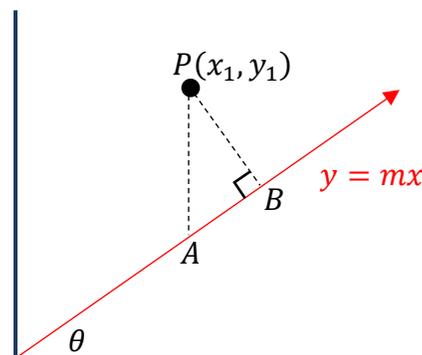
The opposition players can be considered as points on a grid. This is demonstrated in the image on the left.

We need to determine a path that avoids as many players as possible.

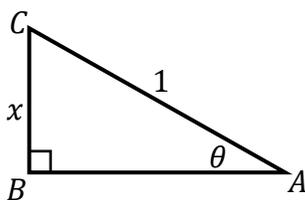
One way (although not the best way) of doing this is to determine the path of the ball that maximizes the sum of the distances from each player to the path.

Consider the diagram on the right showing the path of the ball, and a player located at point P with coordinates (x_1, y_1) . If we assume the ball is hit from the origin then it will follow the line $y = mx$ for some value of m . Point A has the same x -coordinate as point P .

- Determine an expression for θ in terms of m
- Determine an expression for $\angle BPA$ in terms of m
- Determine an expression for distance PA in terms of x_1, y_1 and m
- Determine an expression for distance PB in terms of x_1, y_1 and m



Hint: Expressions such as $\sin(\cos^{-1} x)$, $\tan(\sin^{-1} x)$ etc. can be simplified into expressions which don't contain trigonometric functions. Consider the following example:



- Determine an expression for length AB in terms of x
- Determine an expression for θ in terms of x
- Determine an expression for $\tan(\sin^{-1} x)$ in terms of x

If every player was located above the line $y = mx$ then we could simply repeat the process above for each player and determine the value of m that maximizes the sum of the PB distances. However, some players may be located below the line $y = mx$. In these cases using our answers for the questions above will give a negative value for distance PB . To solve this problem we can square distance PB for each player, and find the value of m that maximizes the sum of the squared distances. You should find you need to maximize the expression

$$\frac{1}{m^2 + 1} \left(\sum y_i^2 - 2m \sum x_i y_i + m^2 \sum x_i^2 \right)$$

Use this as a starting point for an investigation. Extend it in any way you wish...

Criterion C: Communication in Mathematics

Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	<p>The student is able to:</p> <ul style="list-style-type: none"> I. use limited mathematical language II. use limited forms of mathematical representation to present information III. communicate through lines of reasoning that are difficult to interpret. 	<p>The student is able to:</p> <ul style="list-style-type: none"> ○ attempt to explain what is being investigated and how it is being investigated
3 – 4	<p>The student is able to:</p> <ul style="list-style-type: none"> I. use some appropriate mathematical language II. use appropriate forms of mathematical representation to present information adequately III. communicate through lines of reasoning that are complete IV. adequately organize information using a logical structure. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ attempt to explain what is being investigated and how it is being investigated ○ use diagrams to justify some explanations and calculations ○ create a report that is able to be understood without referring to the task sheet
5 – 6	<p>The student is able to:</p> <ul style="list-style-type: none"> I. usually use appropriate mathematical language II. usually use appropriate forms of mathematical representation to present information correctly III. usually move between different forms of mathematical representation IV. communicate through lines of reasoning that are complete and coherent V. present work that is usually organized using a logical structure. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ explain what is being investigated and how it is being investigated (including extending the given scenario) ○ use diagrams to justify explanations and calculations ○ move between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) ○ create a report that is able to be understood without referring to the task sheet
7 – 8	<p>The student is able to:</p> <ul style="list-style-type: none"> I. consistently use appropriate mathematical language II. use appropriate forms of mathematical representation to consistently present information correctly III. move effectively between different forms of mathematical representation IV. communicate through lines of reasoning that are complete, coherent and concise V. present work that is consistently organized using a logical structure 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ clearly explain what is being investigated and how it is being investigated (including extending the given scenario) ○ display formulae clearly and accurately using the equation editor ○ use clear and accurate diagrams to justify explanations and calculations ○ move effectively between explanations, calculations, tables and diagrams with appropriate linking sentences (the following table shows, figure 2 demonstrates etc.) ○ make good use of space on the page (no unnecessary white space, items positioned thoughtfully etc.) ○ create a report that is able to be understood without referring to the task sheet

Criterion D: Applying mathematics in real-life contexts		
Achievement Level	Level Descriptor	Task Specific Clarification
0	The student does not reach a standard described by any of the descriptors below	
1 – 2	<p>The student is able to:</p> <ol style="list-style-type: none"> I. identify some of the elements of the authentic real-life situation II. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ attempt to use appropriate mathematical tools and strategies to investigate the given scenario
3 – 4	<p>The student is able to:</p> <ol style="list-style-type: none"> I. identify the relevant elements of the authentic real-life situation II. select, with some success, adequate mathematical strategies to model the authentic real-life situation III. apply mathematical strategies to reach a solution to the authentic real-life situation IV. discuss whether the solution makes sense in the context of the authentic real-life situation. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ use appropriate mathematical tools and strategies to investigate the given scenario
5 – 6	<p>The student is able to:</p> <ol style="list-style-type: none"> I. identify the relevant elements of the authentic real-life situation II. select adequate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation IV. explain the degree of accuracy of the solution V. explain whether the solution makes sense in the context of the authentic real-life situation. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ use appropriate mathematical tools and strategies to accurately investigate the given scenario ○ give a good attempt at using appropriate mathematical tools and strategies to investigate an extension of the given scenario to a degree of rigour not below that of the given scenario
7 – 8	<p>The student is able to:</p> <ol style="list-style-type: none"> I. identify the relevant elements of the authentic real-life situation II. select appropriate mathematical strategies to model the authentic real-life situation III. apply the selected mathematical strategies to reach a correct solution to the authentic real-life situation IV. justify the degree of accuracy of the solution V. justify whether the solution makes sense in the context of the authentic real-life situation. 	<p>The student is able to</p> <ul style="list-style-type: none"> ○ make thorough use of appropriate mathematical tools and strategies to accurately investigate the given scenario ○ make thorough use of appropriate mathematical tools and strategies to accurately investigate an extension of the given scenario to a degree of rigour not below that of the given scenario