Creating a Casino Game

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Introduction

For casinos around the world, the objective of letting customers play games such as poker or roulette is to gain profit. However, the game must be attractive to the player; in other words, it must seem as though they stand a chance of winning. In order for the casino to make sure that they almost always benefit from the games, Casinos have to be aware of their earnings and payouts; they must be aware of the bank's perspective as well as the player's. Basically, the casino must find a balance between the potential of gaining profit and the customers' incentive to play.

I first gained an interest in probability in games after watching a drama called "Liar Game," in which the main characters must compete in a series of games that are not what they seem. In order for them to win, they had to analyze the probability of them winning depending on who they had as their allies and enemies. The show made me wonder how casinos twist the minds of the customers to make them believe the game is in their favor.

In order to create such a game, mathematical probability becomes relevant. Although players will be able to discover their chance of winning with simple probability, the probability will become difficult to analyze once the number of players, number of spins, and other factors are added to the game.

In this investigation I attempt to create a roulette game that optimizes the profits of the bank but also looks attractive to the player.

Step One: the basics

Firstly, I analyzed the probability of winning in a simple roulette game that I would later build more upon.

This is a basic roulette game that consists of two players. The rules are simple:

- Jane, a player, spins the roulette once and Alex, the casino worker spins once.
- The roulette has six equal divisions, each with a number on it.
- If Jane spins a higher number than the casino, she wins.
- If Alex spins a higher number than Jane or spins the same number, he wins.

The first table shows when Jane will win and the second table shows when Alex will win, depending on what number they spin.

In these cases, Alex will lose

Number Jane spins	Number Alex spins
1	
2	1
3	1,2
4	1,2,3
5	1,2,3,4,
6	1,2,3,4,5

In these cases, Alex will win

Number Jane spins	Number Alex spins
1	1,2,3,4,5,6
2	2,3,4,5,6
3	3,4,5,6
4	4,5,6
5	5,6
6	6

There are 36 possible outcomes in total. This means that the probability that Jane wins is

$$\frac{15}{36} = 41.7\%$$

and the probability that Alex wins is

$$\frac{21}{36} = 58.3\%$$

As we can see from this very basic example, Alex (the casino) has a slightly higher chance of winning than Jane. However, at this stage, the player is able to discern easily his/her chances of winning. But what would happen when extra factors are added to the game?

Step Two: Player spins twice

This time, I experiment with the player being able to spin twice. Jane will choose the higher number from the two spins, but Alex only gets one chance. The probability of Jane winning will probably change.

Let's pretend Alex spins first. He gets 1. Jane has to spin 2,3,4,5, or 6 to win. She has two spins to get a higher number than Alex. In this case, she would have to get 1 on both spins to lose.

If Alex spins a 2, Jane has to spin 3,4,5, or 6 to win. There are 4 possible outcomes with which she would lose: (1,1), (1,2), (2,1), and (2,2).

If Alex spins a 3, there are 9 possible outcomes with which she would lose: (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)

If Alex spins a 4, there are 16 possible outcomes with which she could lose: (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4).

A pattern emerges. Whatever number Alex spins, the number of outcomes with which Jane would lose is the square of that number.

With this pattern, we can find the number of outcomes with which Jane loses for any number Alex spins.

Number Alex spins	Number of outcomes with which Jane loses
1	1
2	4
3	9
4	16
5	25
6	36

Now we can calculate the total probability that Alex wins, and Jane wins.

P(Alex wins) = $\frac{1+4+9+16+25+36}{6\times6\times6} = \frac{91}{216} = 42.1\%$

P(Jane wins) = 1-42.1%=57.9%

Another way to find the probability of Jane losing would be using a formula.

Let *x* be **the number Alex spins**, and *y* be **the number of numbers available for Jane to lose**. The probability Jane would lose is:

$$P(x)\times\left(\frac{y}{6}\right)^2$$

P(x) would always be $\frac{1}{6}$ and *y* would vary. For example, when Alex spins a 2, y would be 2 (because the numbers available for Jane to lose would be 1 and 2. So the probability of Jane losing in this case would be

$$\frac{1}{6}\times\frac{2}{6}\times\frac{2}{6}=\frac{4}{216}$$

Another example would be when Alex spins a 3. *y* would be 3 (numbers available for Jane to lose would be 1,2, and 3). So the probability of Jane losing in this case would be

$$\frac{1}{6}\times\frac{3}{6}\times\frac{3}{6}=\frac{9}{216}$$

From this, a basic formula can be created to show the probability of Jane losing. When *x* is the number Alex spins, the probability of him spinning that number would always be $\frac{1}{6}$. The number of numbers available for Jane to lose, *y*, would always be the same as the number Alex spins (in the case above, Alex spins a 3 and the number of numbers available for Jane to lose is also 3). We also know that *y* has to be squared, as Jane is spinning twice. Therefore, to find the total probability of Jane losing in this game, this formula can be used:

P(Jane loses) = $\sum_{x=1}^{6}\left(\frac{1}{6}\right)\left(\frac{x}{6}\right)^2$

Step Three: Casino also spins twice

What if Alex also spun the roulette twice? The formula would be altered slightly. Let *x* be **Alex's highest number in two spins**, and *y* be **the number of numbers available for Jane to lose**. According to the formula I came up with before in Step two, the probability Jane loses would be:

$$P(x)\times\left(\frac{y}{6}\right)^2$$

However, this time, the probability that Alex spins a certain number would be different unlike the last time when it was just $\frac{1}{6}$. This is because *x* now represents Alex's highest number in two spins instead of just one.

Using examples will help me find a new formula.

Let

x=Alex's highest number in two spins

y= number of numbers available for Jane to lose

a= possible pairs of numbers Alex spins

The probability of variable *x* will always be the number of possible pairs, or the number of *a* values out of 36. For example, when the highest number Alex gets is a 4, there are 7 possible pairs he could have spun, which is the numerator. The denominator, 36, represents the number of all possible pairs.

For example,

When x=1, y=1 and a: (1,1)

$$P(x)\times\left(\frac{y}{6}\right)^2$$

$$\frac{1}{36}\times\left(\frac{1}{6}\right)^2=\frac{1}{1296}$$

When x=2, y=2 and a: (1,2) (2,1) (2,2)

$$P(x)\times\left(\frac{y}{6}\right)^2$$

$$\frac{3}{36}\times\left(\frac{2}{6}\right)^2=\frac{12}{1296}$$

When x=3, y=3 and a: (1,3), (3,1), (2,3), (3,2), (3,3)

$$P(x)\times\left(\frac{y}{6}\right)^2$$

$$\frac{5}{36}\times\left(\frac{3}{6}\right)^2=\frac{45}{1296}$$

When x=4, y=4 and a: (1,4), (4,1), (2,4), (4,2), (3,4), (4,3), (4,4)

$$P(x)\times\left(\frac{y}{6}\right)^2$$

$$\frac{7}{36}\times\left(\frac{4}{6}\right)^2=\frac{112}{1296}$$

When x=5, y=5 and a: (1,5), (5,1), (2,5), (5,2), (3,5), (5,3), (4,5), (5,4), (5,5)

$$P(x)\times\left(\frac{y}{6}\right)^2$$

$$\frac{9}{36}\times\left(\frac{5}{6}\right)^2=\frac{225}{1296}$$

When x=6, y=6 and a: (1,6), (6,1), (2,6), (6,2), (3,6), (6,3), (4,6), (6,4), (5,6), (6,5), (6,6)

$$P(x) \times \left(\frac{y}{6}\right)^2$$

$$\frac{11}{36} \times \left(\frac{6}{6}\right)^2 = \frac{396}{1296}$$

Now we can calculate the total probability that Alex wins, and Jane wins.

P(Alex wins)= $\frac{1}{1296} + \frac{12}{1296} + \frac{45}{1296} + \frac{112}{1296} + \frac{225}{1296} + \frac{396}{1296} = \frac{791}{1296} = 0.6103 = 61\%$

P(Jane wins)= 1-0.6103 =0.3897=39%

Could we have found the possible pairs, or *a*, without actually writing them all down? According to my calculations, this is the number of possible pairs for each value of *x*.

x (highest number Alex spins)	Number of *a* values (number of possible pairs)
1	1
2	3
3	5
4	7
5	9
6	11

Not only are the *a* values all odd, starting from 1, but they also are always equivalent to 2*x*-1.

Therefore, from this observation, a formula can be created to calculate the probability of Jane losing (or Alex winning) in this game. When *x* is the number Alex spins, the probability of Alex getting *x* is the number of possible pairs (2*x*-1) out of 36 pairs. The number of numbers available for Jane to lose is same as the highest number Alex spins, or *x*. Also, $\left(\frac{x}{6}\right)$, or the probability of Jane getting the same or a lower number than Alex, must be squared as Jane spins the roulette twice, same as before. With all this in mind, this is the formula that shows the total probability of Jane losing in this game:

P(Jane loses)= $\sum_{x=1}^{6}\left(\frac{2x-1}{36}\right)\left(\frac{x}{6}\right)^2$

Basically, when both Jane and Alex spin twice and choose the higher number, Jane has 39% chance of winning and Alex has 61%. Compared to when only Jane got to spin twice, the probability of Alex winning has dramatically increased.

At this point, we are tentatively assuming that when Jane spins more times than Alex, her chances of winning are higher.

Step Four: Player spins thrice, Casino twice

I can test my assumption with another game. We can now easily calculate the probability of Jane losing when she spins 3 times and Alex spins 2 times, as we have found most of the needed information previously in Step Three. The probability of Jane losing will change, as the number of times she spins has changed from 2 to 3.

Alex's highest number (*x*)	The probability that Alex spins *x*	Number of numbers available for Jane to lose	Probability that Jane will get the same or a lower number than Alex	Probability Jane loses when Alex spins *x*
1	$\frac{1}{36}$	1	$\left(\frac{1}{6}\right)^3=\frac{1}{216}$	$\left(\frac{1}{36}\right)\left(\frac{1}{216}\right)=\frac{1}{7776}$
2	$\frac{3}{36}$	2	$\left(\frac{2}{6}\right)^3=\frac{8}{216}$	$\left(\frac{3}{36}\right)\left(\frac{8}{216}\right)=\frac{24}{7776}$
3	$\frac{5}{36}$	3	$\left(\frac{3}{6}\right)^3=\frac{27}{216}$	$\left(\frac{5}{36}\right)\left(\frac{27}{216}\right)=\frac{135}{7776}$
4	$\frac{7}{36}$	4	$\left(\frac{4}{6}\right)^3=\frac{64}{216}$	$\left(\frac{7}{36}\right)\left(\frac{64}{216}\right)=\frac{448}{7776}$
5	$\frac{9}{36}$	5	$\left(\frac{5}{6}\right)^3=\frac{125}{216}$	$\left(\frac{9}{36}\right)\left(\frac{125}{216}\right)=\frac{1125}{7776}$
6	$\frac{11}{36}$	6	$\left(\frac{6}{6}\right)^3=\frac{216}{216}$	$\left(\frac{11}{36}\right)\left(\frac{216}{216}\right)=\frac{2376}{7776}$

The total probability that Jane would lose in this game, when adding up all the probabilities above, is

P(Alex wins)= $\frac{4109}{7776}=52.8\%$

P(Jane wins)= 1-52.8%=47.2%

Surprisingly, Jane has a higher chance of losing than winning even when she has 3 spins and Alex has 2. Unlike when she had 2 spins and Alex 1, when she had 42.1 % chance of losing, she now has 52.8% chance of losing.

The assumption I made previously ("We can tentatively assume that when Jane spins more times than Alex, her chances of winning are higher") is the key to this new game. Because Jane has 3 spins, she will assume that she has an advantage over Alex. However, that is not the case mathematically.

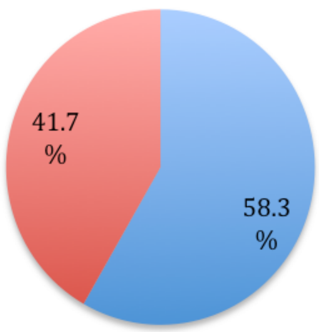
The formula to calculate the probability of Jane losing remains the same as the previous game, but $\frac{x}{6}$, or the probability that Jane gets the same or a lower number than Alex, is cubed instead of squared due to the changed number of spins.

$$P(Jane\ loses) = \sum_{x=1}^{6} \left(\frac{2x-1}{36}\right)\left(\frac{x}{6}\right)^3$$

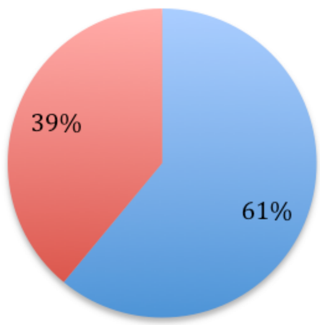
Step Five: Considering the earnings and payouts

Out of all the games I have tested, the last one is the most effective in that it allows the player to think they have an advantage while they actually have a disadvantage. These charts show the probability the player wins in each of the 4 games. Red represents the player winning and blue represents the casino winning.

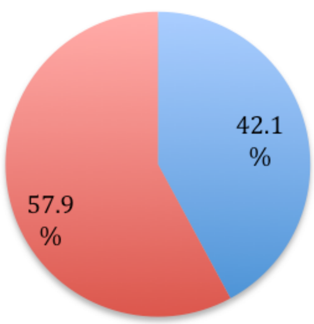
Both spin once



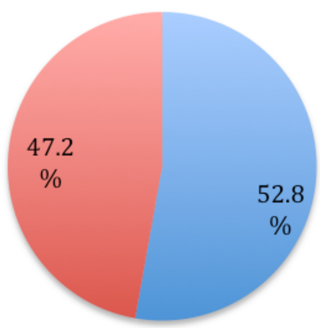
Both spin twice



Player spins twice, casino once



Player spins thrice, casino twice



As anyone can see, the game in which the player spins thrice and the casino twice has the most close to equal probability of both parties winning. This gives the player an illusion that he has a “fair chance” of winning.

Furthermore, unlike when the player and the casino both spin once or both spin twice, the player spins more times than the casino in the fourth game. However, unlike the third game in which the player has an advantage over the casino by spinning more times, the fourth game lets the casino have the advantage.

It is also quite difficult for the player to calculate the probability of himself winning unless they have some time in their hands. As we can see from all the different games for which I calculated the probabilities of the casino and the player winning respectively, the calculation to figure out the probability becomes increasingly complicated when the number of spins increases.

The main point here is that the casino ultimately has to profit from this game.

Let us make the price of the game $10. This is a fairly cheap price that would attract customers.

Would the casino benefit if the player gets back $20 (including the money they paid) if he wins? If the player loses, he loses the $10. The game can be marketed as a quick, simple way to earn some money.

Let’s test it out.

The probability that the casino wins is 52.8%. Therefore, there is 52.8% chance in one game that a casino will win 10$. The other 47.2% will be when the casino loses $10.

This sounds like a fairly equal chance of losing or winning money, but in the long term, will the casino benefit from this game?

Step Six: Calculating the long-term profit

First of all, let’s pretend that 20 people play this game. What would be the casino’s expected profit?

The formula for the casino’s expected profit:

(number of people who have played the game)(0.528)($10) - (number of people who have played the game)(0.472)($10)

$$20 \times 0.528 \times \$10 - 20 \times 0.472 \times \$10$$

$105.6 - $94.4 = $11.2

This does not seem like a large amount of profit, but it will accumulate as more and more people play.

The profits of the casino would be around these values, using the same method as above…

If 1000 people play: $560

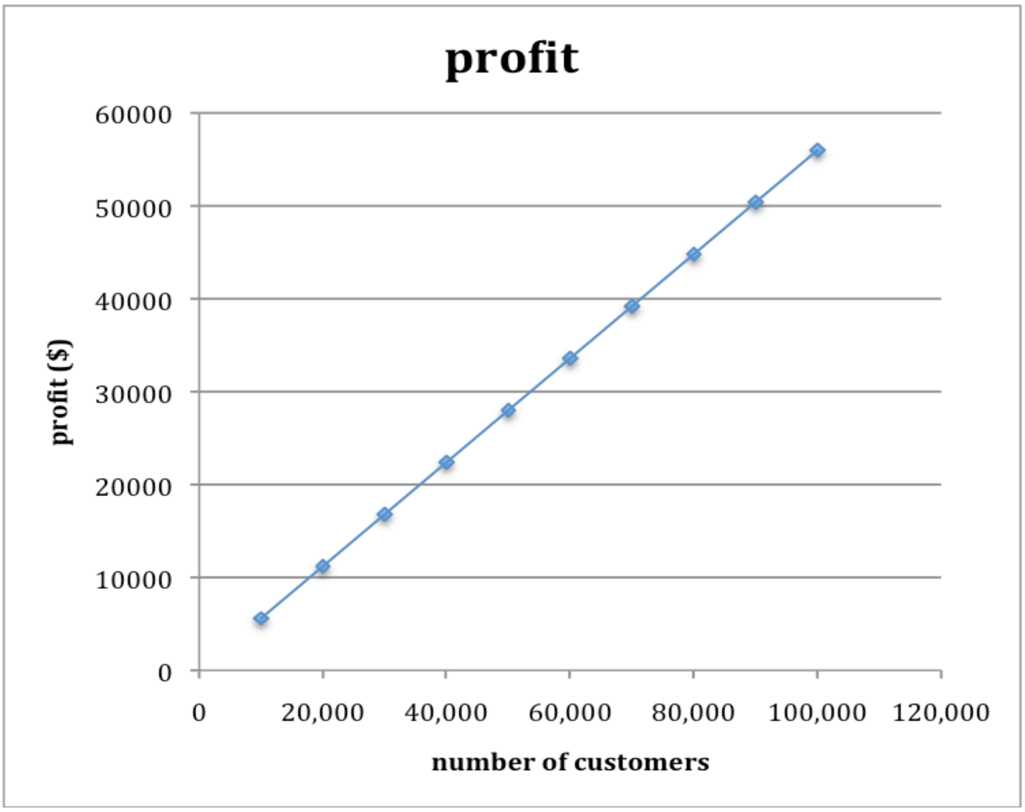
If 50,000 people play: $28000

To give a better understanding of how rapidly the profit would increase, here is a table and a graph of the expected profit every 10,000 people that play:

Customers	Profit ($)
10,000	5600
20,000	11200
30,000	16800
40,000	22400
50,000	28000
60,000	33600
70,000	39200
80,000	44800
90,000	50400
100,000	56000

The profit column shows the total expected profit every time 10,000 more customers have played the game. Therefore, by the time 100,000 people have played the game, the casino would have gained a total profit of 257,600 dollars.

The profit increases in a linear pattern.



Another method I could use to determine how successful the game is using the binomial distribution function. For example, what is the probability that 2 in 3 people will lose the game? This will help show how likely it is that more people will lose than win.

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

$$P(X = 2) = \binom{3}{2} (0.528)^2 (0.472)$$

=0.395 (3s.f)

there is roughly 40% chance that 2/3 people will lose.

Conclusion

Through this investigation, I learned the complex nature of casino games. What I attempted to create was a very simple roulette game, yet required many calculations to have an idea of how much profit it would bring. The complexity of my game was no where near that of an actual roulette game people play in casinos, but calculating the probability took many steps. The trickiest part about creating these games is making the game seem attractive to the customers. The game has to seem relatively fair for the customers to try it. However, I still had to maintain a degree of certainty that the game would benefit the casino in the long term.

The principle behind casino games is similar to that of a crane game I often come across in Japanese game arcades. The stuffed toy has to seem attainable in order for people to keep slotting their coins in the machine; the game has to have an addictive, you-could-win-the-next-time quality.

My game could definitely be improved, and the investigation could be extended. The final model I came up with was still relatively simple, and customers may be able to figure out their disadvantage if they were desperate enough. Increasing the number of divisions on the roulette so there is a wider range of numbers would make the game harder to decipher, with more possible outcomes. Increasing the number of spins and calculating the probability of the player losing for each number of spins would allow me to choose the most superficially attractive game.